Shoreline Profile of Stokes-Mode Edge Waves[a]

Discussion by James T. Kirby,[2] Member, ASCE

The author has shown two interesting results related to progressive nonlinear Stokes-mode edge waves of permanent form. First, it is shown that these waves have a setdown of the mean water level, which is maximum close to shore and decays exponentially fast offshore. However, the presence of this setdown is not apparent in the runup of the edge wave on the beach slope, which, at second order, consists of oscillatory components at the fundamental and second harmonic frequencies with no associated mean. Finally, the form of this shoreline runup is identical to the form of a progressive second-order Stokes wave on deep water, if the vertical plane for the deep-water wave were rotated into the plane of the beach.

In his derivations, the author restricts his results to the case of a beach with small slope β. This choice was perhaps unfortunate, in that it obscures the continuous connection between the forms of the edge-wave solution and the deep-water progressive wave solution. For example, the small-slope solution gives the setdown as

$$\bar{\eta} = -\frac{1}{2} kA^2 \sin \beta e^{-2ky\cos\beta} \quad \text{(14)}$$

Since the factor $\sin \beta$ is linked to the shoreline runup amplitude, it is apparent that there is no remaining slope dependence in the setdown. And yet, it is clear that the setdown should vanish identically as the beach plane is rotated to a vertical position (i.e., as $\beta \rightarrow \pi/2$), in order for the deep-water profile (which would coincide with the runup on a vertical wall oriented parallel to the direction of propagation) to have no mean shift. Ths connection is pursued in this discussion.

First, a solution for the second-order Stokes edge wave on a beach of arbitrary slope is sought in Eulerian coordinates. Following the notation used by the author, the result may be written as

$$\eta = A \sin \beta \cdot \left\{ e^{-kY} \cos(kx - \omega t) + \frac{kA}{2} [\sin^2\beta \cos(2kx - 2\omega t) - \cos^2\beta] e^{-2kY} \right\} \quad \text{(15)}$$

where, in the plane of the still water surface, $Y = y \cos \beta$. The mean water level component is seen to have a $\cos^2\beta$ dependence and hence vanishes as the beach face approaches the vertical, as expected.

The shoreline runup is obtained by means of a Taylor expansion, which was verified by the author. Denote the instantaneous shoreline position along the beach face as Y_{sh}, measured positive from the still water shoreline. In unrotated coordinates, an expression for the elevation of the shoreline is given by

$$\eta_{sh} = \eta(y = 0) + \frac{\partial \eta}{\partial y}(y = 0) y_{sh} \quad \text{(16)}$$

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where y_{sh} = the horizontal distance to the instantaneous shoreline from the still water shoreline. Using some geometry, we obtain

$$Y_{sh} \sin \beta = \eta(Y = 0) + \cos^2\beta \frac{\partial \eta}{\partial Y}(Y = 0)Y_{sh} \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Solving this expression for Y_{sh} gives finally

$$Y_{sh} = -A \left[\cos(kx - \omega t) + \frac{1}{2} kA \cos(2kx - 2\omega t)\right] + O(A)^3 \quad \ldots\ldots (18)$$

which is the desired result. Note that the expression for $\eta(Y = 0)$ is asymptotic to $-Y_{sh}$ as $\beta \rightarrow \pi/2$, which is the desired result.

It is noted in closing that the results obtained by the author would be adequate for all practical purposes; indeed, the same expression for the shoreline runup, which is invariant under changes in slope, is obtained. However, the present result provides a more complete exposition of the theory.