Carmen, P. C. (1937). *Trans.*, Institute of Chemical Engineers (London), 15, 150.
den Adel, H. (1987). "Cyclic permeability of granular material." *Delft Soil Mechanics Report No. CO-272550/35,* Delft Soil Mechanics, Delft, Holland.
Dudgeon, C. R. (1964). "Flow of water through coarse granular material," ME thesis, University of New South Wales, Sydney, Australis.
Engelund, F. (1953). *Trans.*, Danish Academy of Technical Science, 3, 1.
Ergun, S. (1952). "Fluid flow through packed columns." *Chem. Eng. Prog.*, 48, 89–94.
Hall, K. R. (1987). "A study of the stability of rubblemound breakwaters," PhD thesis, University of New South Wales, Sydney, Australia.
Hannoura, A. A., and McCorquodale, J. A. (1978). "Virtual mass of coarse granular media." *J. Wtrwy., Port, Coast. and Oc. Div.*, ASCE, 104(2), 191–200.
Hannoura, A. A., and McCorquodale, J. A. (1985). "Rubble mounds: hydraulic conductivity equation." *J. Wtrway., Port, Coast. and Oc. Div.*, ASCE, 111(5), 783–816.
Muskat, M. (1937). *The flow of homogeneous fluids through porous media,* Edwards Publishing, Ann Arbor, Mich.
Scheidegger, A. (1960). *Physics of flow in porous media.* University of Toronto Press, Toronto, Canada.
Smith, G. (1991). "Comparison of stationary and oscillatory flow through porous media," MS thesis, Queen's University, Kingston, Ontario.
Ward, J. C. (1964). "Turbulent flow in porous media." *J. Hydr. Div.*, ASCE, 90(90), 1–11.

Linear Surface Waves over Rotating Fluids[a]

Discussion by James T. Kirby,[2] Member, ASCE

The author has presented what is purported to be a model for both short and long linear waves in a slowly varying, rotating domain. In the short-wave, high-frequency limit with negligible rotation effects, the model equation recovers the mild-slope equation of Berkhoff (1972). However, in the low frequency limit, the asymptotic form of the model reduces to

$$\nabla\cdot(gh\nabla\eta) + \omega^2\left[1 - \left(\frac{f}{\omega}\right)^2\right]\eta = 0 \quad (51)$$

This form is not a valid asymptote for the varying depth case. For the case of long waves in a rotating domain, the governing wave equation is given by

$$\eta_{ttt} + f^2\eta_t - g\nabla\cdot(h\nabla\eta_t) - gfJ(h, \eta) = 0 \quad (52)$$

where J = the Jacobian operator:

$$J(h, \eta) = h_x\eta_y - h_y\eta_x \quad (53)$$

This equation follows from eliminating the velocities from the long wave

[a]March/April, 1991, Vol. 117, No. 2, by Ting-Kuei Tsay (Paper 25662).
[2]Assoc. Prof., Ctr. for Appl. Coastal Res., Dept. of Civ. Engrg., Univ. of Delaware, Newark, DE 19716.

equations, and can be found in standard texts [see, for example, Pedlosky (1979)]. For the case of time-harmonic motions:

$$\eta(x, y, t) = \eta(x, y)e^{-i\omega t} \qquad (54)$$

(52) reduces to

$$\nabla \cdot (gh\nabla\eta) + \omega^2 \left[1 - \left(\frac{f}{\omega}\right)^2\right] \eta + ig \left(\frac{f}{\omega}\right) J(h, \eta) = 0 \qquad (55)$$

It is clear that (55) differs from (51) in the inclusion of the Jacobian term.

Since the author's primary example is for the case of tidal waves being scattered by an island with surrounding topography, the neglect of the Jacobian term puts the validity of the results in question. In particular, all other things being equal, the Jacobian term approaches zero like (f/ω) as ω increases, whereas the effect on the wavelength through the remaining rotational term (and in the dispersion relation) vanishes more quickly, like $(f/\omega)^2$. It is of some interest to specify how small the bottom slope terms in the Jacobian would have to be before that term could be neglected in comparison with the remaining rotational term. Consequently, let L characterize a wavelength, R and H characterize horizontal and vertical topographic length scales, and a characterize a wave amplitude. We are interested in the value of the ratio

$$N = \left[\frac{g\left(\frac{f}{\omega}\right) J(h, \eta)}{\left(\frac{f}{\omega}\right)^2 \omega^2\eta}\right] \qquad (56)$$

and require $N << 1$ for the Jacobian term to be negligible. Substituting the characteristic dimensions and using the dispersion relation (19b) together with

$$\omega^2 = g\lambda^2 h \qquad (57)$$

for long Poincaré waves, leads to

$$N = \frac{L}{R}\left(\frac{f}{\omega}\right)^{-1} \qquad (58)$$

For the cases where rotation is important, $f/\omega = O(1)$, and so we require $L/R << 1$.

To use one of the author's computations as an example, we consider the case shown in Fig. 2. Correspondingly, we take the tidal period $T = 6$ hours, $f = 10^{-4}\ s^{-1}$, $H = 4$ km, the ocean depth, and $R = 1{,}500$ km, the radius of the island. For these parameters, we get $f/\omega = 0.344$ and hence rotational effects are fairly weak. The wavelength $L = 4{,}555$ km. The parameter N then has a value $N = 8.83$, which is far too large to justify the neglect of the Jacobian term.

Finally, we remark that the model equation presented here would also be invalid in the combined limit of high-frequency and rapid rotation for a frame on a rotating sphere, although this combined limit has no geophysical significance. The full set of acceleration terms for the general three-dimensional case is given by (Daily and Harleman 1966)

$$\frac{du}{dt} - fv + 2\Omega \cos \psi w \quad (59)$$

$$\frac{dv}{dt} + fu \quad (60)$$

$$\frac{dw}{dt} - 2\Omega \cos \psi u \quad (61)$$

in contrast to the set given in (2)–(4). (Note that centripetal terms have been absorbed in the hydrostatic pressure distribution.) The neglect of the w term in (59) to obtain (2) is usually justified by assuming that the vertical motions are weak relative to horizontal motions. This assumption is not valid for the fully dispersive waves being considered. The use of this assumption reduces the vertical momentum balance to the hydrostatic relationship; this step has not been taken. There is then no scaling argument which could remove the remaining u term in the vertical momentum balance, whereas it has been neglected by the author.

Appendix. References

Daily, J. W., and Harleman, D. R. F. (1966). *Fluid dynamics,* Addison-Wesley, Reading, Mass.

Pedlosky, J. (1979). *Geophysical fluid dynamics,* Springer Verlag, New York, N.Y.

Discussion by F. Mattioli[3]

The author derives an equation for the propagation of linear waves in a rotating fluid, using an approach similar to that followed by Berkhoff (1976) to derive the so-called mild slope equation for intermediate-depth waves. However, his equation is different with respect to the equation derived by Mattioli (1981a, b) for the same problem, but in the limit case of shallow water.

In that case the starting equations were

$$-\iota\omega \mathbf{u} + f\hat{\mathbf{k}} \times \mathbf{u} + g\nabla\zeta = 0 \quad (62)$$

$$-\iota\omega\zeta + \nabla \cdot (h\mathbf{u}) = 0 \quad (63)$$

where ζ = the surface elevation, $\mathbf{u}$ = the horizontal velocity, ω = the angular frequency, $\hat{\mathbf{k}}$ the versor upwardly directed, and h = the water depth. After suitable manipulation of the equations, one obtains

$$\nabla \cdot (h\nabla_m\zeta) + \frac{\omega^2 - f^2}{g}\zeta = 0 \quad (64)$$

with the boundary conditions

$$\partial_m\zeta = -\frac{\omega^2 - f^2}{\iota\omega g}\hat{\mathbf{n}} \cdot \mathbf{u} \quad (65)$$

where $\hat{\mathbf{n}}$ = the unit versor outwardly directed along the boundary of the basin.

[3]Dipartimento di Fiscia, Via Irnerio 46, Università di Bologna, Bologna, Italy.