Spectral Evolution of Nonlinear Directional Waves in Shallow Water

James M. Kaihatu[1] and James T. Kirby[2]

Introduction

As waves propagate over the coastal shelf into shallow water, they undergo significant change due to bathymetric effects, wave-current interaction, and nonlinear energy transfer. Evolution of waves in shallow water can be modeled using the standard Boussinesq equations of Peregrine (1967) or, in frequency-domain form, the models of Freilich and Guza (1984), Liu et al. (1985), and Kirby (1990). However, there may be a range of frequencies which are in intermediate or deep water for a given spectrum, violating the small kh (where k is the wavenumber and h the water depth) assumption inherent in the classical Boussinesq equations.

Recently Agnon et al. (1993) and Kaihatu and Kirby (1995) have developed nonlinear wave transformation models which make use of the full dispersive effects of linear theory. This allows the linear propagation of the entire spectral frequency range to be modeled. Neither formulation, however, is suitable for modeling nearshore evolution of directional spectra. This is because these models have interaction coefficients which cannot discriminate between waves of the same frequency but different direction; thus the cross-spectral energy transfer between waves approaching at different angles cannot be weighted by this angular separation.

In this study we will develop an angular spectrum model from the theory of Kaihatu and Kirby (1995). The linear properties of the resulting model will have no kh restriction, and we will also be able to simulate the transfer of energy between waves with an angular separation. The initial development (outlined below) assumes longshore homogeneity of water depth; however we will extend the development to account for variations of the water depth in the longshore direction (Dalrymple et al. 1989; Suh et al. 1990) as well as the presence of a spatially-varying ambient current field.

Dispersive Angular Spectrum Model

We can develop the angular spectrum model from Equation 22 of Kaihatu and Kirby (1995). Instead of applying the parabolic approximation to this time-periodic equation, we make the assumption of longshore (y) periodicity:

$$\hat{\phi}_n = \sum_{m=-M}^{M} A_n^m e^{im\lambda y + \int k_n \gamma_n^m dx} \qquad (1)$$

where $\lambda = k\sin\theta$ and $\gamma^m{}_n = \cos\theta$. After neglecting y-variations in water depth (though these can be straightforwardly treated) we obtain:

[1] Oceanographer, Ocean Dynamics and Prediction Branch, Oceanography Division (Code 7322), Naval Research Laboratory, Stennis Space Center, MS 39529-5004.
[2] Professor, Center for Applied Coastal Research, University of Delaware, Newark, DE 19716.

$$
2ik_n\gamma_m^m (CC_g)_n A_{n,x}^m + i(k_n\gamma_n^m (CC_g)_n)_x A_n^m + (k_n^2(1-\gamma_n^{m2}) - m^2\lambda^2)(CC_g)_n A_n^m
$$
$$
= \frac{i}{4}\left[\sum_{l=1}^{n-1}\sum_{p=-M}^{M} I_{n,l}^{m,p} A_l^p A_{n-l}^{m-p} e^{i\int k_l\gamma_l^p + k_{n-l}\gamma_{n-l}^{m-p} - k_n\gamma_n^m dx} + 2\sum_{l=1}^{N-n}\sum_{p=-M}^{M} J_{n,l}^{m,p} A_l^{*p} A_{n+l}^{m+p} e^{i\int k_{n+l}\gamma_{n+l}^{m+p} - k_l\gamma_l^p - k_n\gamma_n^m dx}\right] \quad (2)
$$

where I and J are interaction coefficients. This formulation allows the directional interactions in spectra to be modeled, since each component in an (f,θ) directional spectrum occupies a unique bin in (f,λ) space. We will also extend this formulation to account for longshore nonhomogeneity in the water depth and interaction with spatially-varying current fields.

Stochastic Formulation

Much recent work on shallow water modeling has involved developing "phase averaged" formulations of triad interactions. Examples include the shallow water formulation of Abreau et al. (1992) and the parameterized triad formulation of Eldeberky and Battjes (1995). We intend to develop averaged equations for wave transformation with triad interactions, with the aim of examining closely the nature of the closure assumptions used in the derivation.

References

Abreau, M., A. Larraza and E. B. Thornton, "Nonlinear transformation of directional spectra in shallow water," *J. Geophys. Res.*, **97**, 15579-15589, 1992.

Agnon, Y., A. Sheremet, J. Gonsalves and M. Stiassnie, "Nonlinear evolution of a unidirectional shoaling wave field," *Coast. Eng.*, **20**, 29-58, 1993.

Dalrymple, R.A., K.D. Suh, J.T. Kirby and J.W. Chae, "Models for very wide-angle water waves and wave diffraction. Part 2. Irregular bathymetry," *J. Fluid Mech.*, **201**, 299-322, 1989.

Eldeberky, Y., and J. Battjes, "Parameterizations of triad interactions in wave energy models," *Proc. Coastal Dynamics Conf.*, Gdansk, Poland, 140-148, 1995.

Freilich, M.H., and R.T. Guza, "Nonlinear effects on shoaling surface gravity waves," *Phil. Trans., Roy. Soc. of London*, **A311**, 1-41, 1984.

Kaihatu, J.M., and J.T. Kirby, "Nonlinear transformation of waves in finite water depth," *Phys. Fluids*, **7**, 1903-1914, 1995.

Kirby, J.T., "Modeling shoaling directional wave spectra in shallow water," *Proc., 22nd Inter. Conf. On Coast. Eng,* Delft, the Netherlands, 109-121, 1990.

Liu, P.L.-F., S.B. Yoon and J.T. Kirby, "Nonlinear refraction-diffraction of waves in shallow water," *J. Fluid Mech.*, **153**, 185-201, 1985.

Peregrine, D.H., "Long waves on a beach," *J. Fluid Mech.* **27**, 815-827, 1967.

Suh, K.D., R.A. Dalrymple and J.T. Kirby, "An angular spectrum model for propagation of Stokes waves," *J. Fluid Mech.*, **221**, 205-232, 1990.