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A source function method for generation of waves on currents in Boussinesq models

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Abstract

A source function method for the generation of waves internal to Boussinesq model grid boundaries (Wei G, Kirby JT, Sinha A. Generation of waves in Boussinesq models using a source function method. Coastal Engng 1999;36:271–299) is generalized to account for the presence of a strong current, for application to wave-current interaction problems. The method is also modified to eliminate waves propagating backwards from the source region. The resulting modification greatly reduces the extent of sponge layers or other absorbing layers needed on the upwave open boundary in Boussinesq model applications to coastal regions, where shoreline reflection is weak.

Keywords: Source function method; Wave generation; Boussinesq model

1. Introduction

The need to propagate waves across the seaward boundary of a model grid in a Boussinesq model simulation presents us with a difficult problem, since the well-posedness of the open boundary condition is often not established and the treatment of radiating waves arriving from inside the domain is thus not well defined. Consequently, this problem is often handled quite empirically, using a source function method to generate waves in the grid interior, coupled with the use of extensive absorbing sponge layers to remove both unwanted backward-propagating waves from the source as well as reflected waves radiating out of the numerically-simulated region. This method was pioneered by Larsen and Dancy [5], and can be made to work quite effectively if the wave-absorbing properties of the sponge layer seaward of the source are treated carefully. The method has recently been extended to the case of a spatially-distributed source of either mass or momentum, and a linearized solution has been provided for Boussinesq models with $O(kh)^2$ dispersion by Wei et al. [11]. Gobbi and Kirby [3] (hereafter referred to as GK99) have provided the extension of the linearized result to models with $O(kh)^4$ dispersion and corresponding higher-order spatial derivatives (up to 5th order).

The advantages of using a spatially distributed source function are that it is simple to implement and the source is independent of the geometry of the absorber. However, in all the cases mentioned above, the wave source generates both forward- and backward-propagating waves, and the presence of an extensive and carefully designed sponge layer behind the source region is thus required. This problem can be handled quite adequately, but the resulting damping region can be large and thus adds measurably to the computational effort required during model simulations. In many applications to coastal regions, shoreline reflection of incident wind waves is quite weak, and thus most of the action of the damping region behind the source is dedicated to eliminating the spurious backward-propagating wave. In this note, we suggest an alternate formulation which uses a mass and momentum source in tandem, and which eliminates the backward-propagating component of the generated wave in the linear approximation. The resulting generation mechanism is tested in the model of Wei et al. [10], hereafter referred to as WKGS.

As pointed out by Kirby [4], Madsen et al. [6] and Chen et al. [2], models such as WKGS which fall in the "fully-nonlinear" category are capable of correctly simulating the dispersion of short waves riding on strong currents, as

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the proper combination of terms giving a total derivative following the current **U**,

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \mathbf{U}\cdot\nabla \tag{1}$$

is retained in all higher-order dispersive terms. We therefore extend the method here to include the effect of a specified O(1) current field, and test the resulting model for cases involving steady currents. In the following section, we present the extensions to the $O(kh)^2$ WKGS model. The more extensive development for the $O(kh)^4$ model of GK99 is given in Appendix A.

2. Theory for the $O(kh)^2$ model

The WKGS model consists of a set of fully nonlinear Boussinesq-type equations which simulate wave propagation. The governing equations in non-dimensional form are given by

$$\eta_t + \nabla\cdot((h+\delta\eta)\mathbf{u}) + \nabla\cdot\left[\mu^2\left\{\left[\frac{z_\alpha^2}{2} - \tfrac{1}{6}(h^2 - h\delta\eta + (\delta\eta)^2)\right]\nabla(\nabla\cdot\mathbf{u}) + [z_\alpha + \tfrac{1}{2}(h-\delta\eta)]\nabla(\nabla\cdot(h\mathbf{u}))\right\}\right] = 0 \tag{2a}$$

$$\begin{aligned}\mathbf{u}_t &+ \delta(\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla\eta + \mu^2\left[\frac{z_\alpha^2}{2}\nabla(\nabla\cdot\mathbf{u}_t) + z_\alpha\nabla(\nabla\cdot(h\mathbf{u}_t)) - \nabla\left[\frac{(\delta\eta)^2}{2}\nabla\cdot\mathbf{u}_t + \delta\eta\nabla\cdot(h\mathbf{u}_t)\right]\right] \\ &+ \delta\mu^2\left[\nabla\left[(z_\alpha - \delta\eta)(\mathbf{u}\cdot\nabla)(\nabla\cdot(h\mathbf{u})) + \tfrac{1}{2}(z_\alpha^2 - (\delta\eta)^2)(\mathbf{u}\cdot\nabla)(\nabla\cdot\mathbf{u})\right] + \tfrac{1}{2}\nabla[(\nabla\cdot(h\mathbf{u}) + \delta\eta\nabla\cdot\mathbf{u})^2]\right] = 0\end{aligned} \tag{2b}$$

where

$$\nabla \equiv \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y}$$

and where $\mu = kh$, and $\delta = a/h$ are the dispersive and nonlinearity parameters, respectively. Since the linear, constant-depth form of the governing equations can be explicitly solved (using Green's functions) to obtain a transfer function between the source and the desired wave, we assume linearity ($\delta \ll 1$) and constant water depth. Furthermore, since we are including O(1) currents, we define the total velocity **u** by

$$\mathbf{u} = \frac{1}{\delta}\mathbf{U} + \mathbf{u}_\mathrm{w} \tag{3}$$

where $\mathbf{u}_\mathrm{w}$ is the velocity due to wave motion. The current velocity **U** is also assumed constant. Substituting Eq. (3) in Eq. (2) and keeping terms up to $O(\delta^0)$, we get

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} + h\nabla\cdot\mathbf{u}_\mathrm{w} + h^3\mu^2\nabla\cdot[(\alpha + \tfrac{1}{3})\nabla(\nabla\cdot\mathbf{u}_\mathrm{w})] = 0 \tag{4a}$$

$$\frac{\mathrm{d}\mathbf{u}_\mathrm{w}}{\mathrm{d}t} + \nabla\eta + \mu^2\alpha h^2\frac{\mathrm{d}}{\mathrm{d}t}(\nabla(\nabla\cdot\mathbf{u}_\mathrm{w})) = 0 \tag{4b}$$

where

$$\alpha \equiv \frac{1}{2}\left(\frac{z_\alpha}{h}\right)^2 + \frac{z_\alpha}{h}$$

z_α is a reference depth where $\mathbf{u}_\mathrm{w}$ is defined, and is chosen to obtain appropriate dispersion characteristics even in deep water (see WKGS or Nwogu [7]).

Writing Eq. (4) in dimensional form and introducing source functions in both the continuity and momentum equations we get

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} + h\nabla\cdot\mathbf{u}_\mathrm{w} + \alpha_1 h^3\nabla^2(\nabla\cdot\mathbf{u}_\mathrm{w}) = f(x,y,t) \tag{5a}$$

$$\frac{\mathrm{d}\mathbf{u}_\mathrm{w}}{\mathrm{d}t} + g\nabla\eta + \alpha h^2\nabla^2\left(\frac{\mathrm{d}\mathbf{u}_\mathrm{w}}{\mathrm{d}t}\right) = -g\nabla P(x,y,t) \tag{5b}$$

where $\alpha_1 \equiv \alpha + (1/3)$. Introducing a velocity potential $\phi(x,y,t)$ for $\mathbf{u}_\mathrm{w}$, we can rewrite Eq. (5) as one equation:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}t^2} - gh\nabla^2\phi - \alpha_1 gh^3\nabla^2(\nabla^2\phi) + \alpha h^2\nabla^2\left(\frac{\mathrm{d}^2\phi}{\mathrm{d}t^2}\right) = -g\left(f + \frac{\mathrm{d}P}{\mathrm{d}t}\right) \tag{6}$$

Taking a Fourier transform in the y spatial direction

$$\phi(x,y,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{\phi}(x,\lambda,t)\exp(\mathrm{i}\lambda y)\,\mathrm{d}\lambda \tag{7a}$$

$$f(x,y,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{f}(x,\lambda,t)\exp(\mathrm{i}\lambda y)\,\mathrm{d}\lambda \tag{7b}$$

$$P(x,y,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{P}(x,\lambda,t)\exp(\mathrm{i}\lambda y)\,\mathrm{d}\lambda \tag{7c}$$

and subsequently a Fourier transform in t

$$\tilde{\phi}(x,\lambda,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{\phi}(x,\lambda,\omega)\exp(-\mathrm{i}\omega t)\,\mathrm{d}\omega \tag{8a}$$

$$\tilde{f}(x,\lambda,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{f}(x,\lambda,\omega)\exp(-\mathrm{i}\omega t)\,\mathrm{d}\omega \tag{8b}$$

$$\tilde{P}(x,\lambda,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{P}(x,\lambda,\omega)\exp(-\mathrm{i}\omega t)\,\mathrm{d}\omega \tag{8c}$$

yields a fourth-order ordinary differential equation

$$A\hat{\phi}^{(4)} + B\hat{\phi}^{(3)} + C\hat{\phi}^{(2)} + D\hat{\phi}^{(1)} + E\hat{\phi} = g(\hat{f} + U\hat{P}^{(1)} - \mathrm{i}(\omega - \lambda V)\hat{P}) \quad (9)$$

where $(\ )^{(n)} \equiv (\mathrm{d}^n/\mathrm{d}x^n)(\ )$, and $\mathbf{U} = \mathbf{i}U + \mathbf{j}V$. The constants are given by

$$A = \alpha_1 gh^3 - \alpha h^2 U^2 \quad (10a)$$

$$B = 2\mathrm{i}(\omega - \lambda V)U\alpha h^2 \quad (10b)$$

$$C = gh + (\omega - \lambda V)^2\alpha h^2 + \lambda^2(U^2\alpha h^2 - 2\alpha_1 gh^3) - U^2 \quad (10c)$$

$$D = 2\mathrm{i}U(\omega - \lambda V)[1 - \alpha(\lambda h)^2] \quad (10d)$$

$$E = (\omega - \lambda V)^2[1 - \alpha(\lambda h)^2] - gh\lambda^2[1 - \alpha_1(\lambda h)^2] \quad (10e)$$

The homogenous solution for Eq. (9) is of the form

$$\hat{\phi} \sim \exp(\mathrm{i}lx)$$

where l satisfies the Doppler-shifted dispersion relation of the Boussinesq equations

$$(\omega - \lambda V - lU)^2 = gh(\lambda^2 + l^2)\frac{[1 - \alpha_1 h^2(\lambda^2 + l^2)]}{[1 - \alpha h^2(\lambda^2 + l^2)]} \quad (11)$$

To obtain a particular solution for $\hat{\phi}$, and subsequently a relationship between wave amplitude and source function amplitudes, we use the method of Green's functions.

2.1. Green's function

Consider a Green's function $G(\zeta, x)$, such that

$$A\frac{\partial^4 G(\zeta,x)}{\partial x^4} - B\frac{\partial^3 G(\zeta,x)}{\partial x^3} + C\frac{\partial^2 G(\zeta,x)}{\partial x^2} - D\frac{\partial G(\zeta,x)}{\partial x} + EG(\zeta,x) = \delta(\zeta - x) \quad (12)$$

where $\delta(\zeta - x)$ is a delta function.

Imposing radiating boundary conditions we can write $G(\zeta, x)$ in the form of the homogeneous solution for $\hat{\phi}$ in the region $x \neq \zeta$

$$G(\zeta,x) = \tilde{a}\exp[\mathrm{i}\tilde{l}(x - \zeta)] \qquad \text{for } x > \zeta \quad (13a)$$

$$G(\zeta,x) = a\exp[\mathrm{i}l(\zeta - x)] \qquad \text{for } x < \zeta \quad (13b)$$

where l satisfies Eq. (11), and $\tilde{l}$ satisfies the same equation but with the opposite sign for U. Substituting in Eq. (12) we get

$$A\tilde{l}^4 + \mathrm{i}B\tilde{l}^3 - C\tilde{l}^2 - \mathrm{i}D\tilde{l} + E = 0 \qquad \text{for } x > \zeta \quad (14a)$$

$$Al^4 - \mathrm{i}Bl^3 - Cl^2 + \mathrm{i}Dl + E = 0 \qquad \text{for } x < \zeta \quad (14b)$$

In the absence of currents Eq. (14) gives two roots

$$l_1 = \tilde{l}_1 = \left(\frac{C - \sqrt{C^2 - 4AE}}{2A}\right)^{1/2} \quad (15a)$$

$$l_2 = \tilde{l}_2 = \left(\frac{C + \sqrt{C^2 - 4AE}}{2A}\right)^{1/2} \quad (15b)$$

where l_1 is a real root and l_2 is an imaginary root.

In the presence of currents there are two additional distinct roots in each case. These roots correspond to waves arising due to reflection on a current (Peregrine [8]) and can be ignored as part of the generation problem. Thus, in the presence of currents Eq. (14) is solved numerically using the Newton–Raphson technique with Eq. (15) as initial conditions to obtain a real and an imaginary root. We can therefore construct the Green's function as

$$G(\zeta,x) = \tilde{a}_1\exp[\mathrm{i}\tilde{l}_1(x - \zeta)] + \tilde{a}_2\exp[\mathrm{i}\tilde{l}_2(x - \zeta)] \quad \text{for } x > \zeta \quad (16a)$$

$$G(\zeta,x) = a_1\exp[\mathrm{i}l_1(\zeta - x)] + a_2\exp[\mathrm{i}l_2(\zeta - x)] \quad \text{for } x < \zeta \quad (16b)$$

Integrating Eq. (12) from $x = \zeta - 0$ to $x = \zeta + 0$, and assuming continuity of G, $\partial G/\partial x$, and $\partial^2 G/\partial x^2$, we get the following set of matching conditions

$$G\big|_{\zeta-0}^{\zeta+0} = 0 \quad (17a)$$

$$\frac{\partial G}{\partial x}\bigg|_{\zeta-0}^{\zeta+0} = 0 \quad (17b)$$

$$\frac{\partial^2 G}{\partial x^2}\bigg|_{\zeta-0}^{\zeta+0} = 0 \quad (17c)$$

$$\frac{\partial^3 G}{\partial x^3}\bigg|_{\zeta-0}^{\zeta+0} = \frac{1}{A} \quad (17d)$$

Substituting Eq. (16) in Eq. (17) and solving gives

$$\tilde{a}_1 = \frac{\mathrm{i}}{A(\tilde{l}_1 - \tilde{l}_2)(\tilde{l}_1 + l_1)(\tilde{l}_1 + l_2)} \quad (18a)$$

$$\tilde{a}_2 = \frac{\mathrm{i}}{A(\tilde{l}_2 - \tilde{l}_1)(\tilde{l}_2 + l_1)(\tilde{l}_2 + l_2)} \quad (18b)$$

$$a_1 = \frac{\mathrm{i}}{A(l_1 - l_2)(l_1 + \tilde{l}_1)(l_1 + \tilde{l}_2)} \quad (18c)$$

$$a_2 = \frac{\mathrm{i}}{A(l_2 - l_1)(l_2 + \tilde{l}_1)(l_2 + \tilde{l}_2)} \quad (18d)$$

Eq. (16) together with Eq. (18) gives a Green's function $G(\zeta, x)$ which satisfies Eq. (12) over the entire domain.

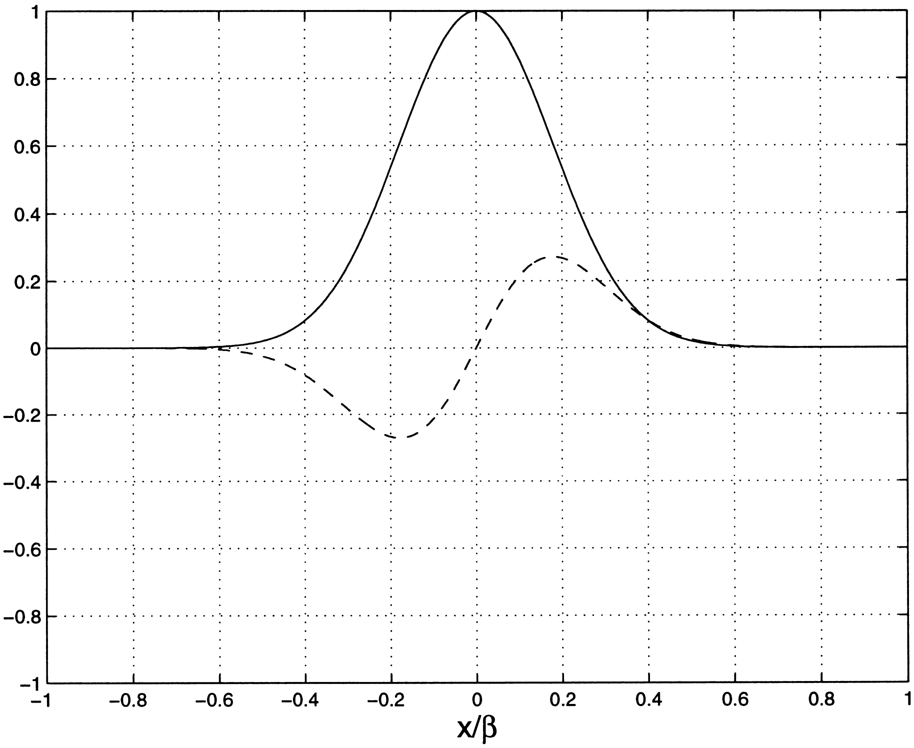

Fig. 1. Source as a function of x. Solid line corresponds to $\hat{f}(x)/D_1$ and dashed line corresponds to $\hat{P}(x)/D_2$.

2.2. Solution for $\hat{\phi}$

We can now solve for $\hat{\phi}$ in terms of the known function $G(\zeta, x)$ and consequently obtain the required source function such that waves of a desired wave amplitude move in one direction only.

Multiplying $G(\zeta, x)$ with Eq. (9), integrating from $x = -\infty$ to $x = \infty$, and using Eq. (12) together with the properties of the delta function gives

$$\hat{\phi}(\zeta,\omega,\lambda) = g\int_{-\infty}^{\zeta} G\left(\hat{f} + U\frac{\partial \hat{P}}{\partial x} - \mathrm{i}(\omega - \lambda V)\hat{P}\right)\mathrm{d}x + g\int_{\zeta}^{\infty} G\left(\hat{f} + U\frac{\partial \hat{P}}{\partial x} - \mathrm{i}(\omega - \lambda V)\hat{P}\right)\mathrm{d}x - \hat{\phi}\frac{\partial^2 G}{\partial x^2}\bigg|_{-\infty}^{\infty} - \hat{\phi}G|_{-\infty}^{\infty} \quad (19)$$

In the absence of a current the boundary terms in Eq. (19) cancel out. In our solution we shall neglect these boundary terms by assumption, and then demonstrate later that the resulting solution exhibits the expected behavior.

For large positive values of ζ the solution approaches

$$\hat{\phi}(\zeta,\omega,\lambda) = g\int_{-\infty}^{\zeta} G\left(\hat{f} + U\frac{\partial \hat{P}}{\partial x} - \mathrm{i}(\omega - \lambda V)\hat{P}\right)\mathrm{d}x \quad (20)$$

Substituting Eq. (16) in Eq. (20) and noting that l_2 being imaginary leads to exponentially decaying terms, we get

$$\hat{\phi}(\zeta,\omega,\lambda) \approx g a_1 \exp(\mathrm{i}l_1\zeta)\int_{-\infty}^{\infty}\left(\hat{f} + U\frac{\partial \hat{P}}{\partial x} - \mathrm{i}(\omega - \lambda V)\hat{P}\right) \times \exp(-\mathrm{i}l_1 x)\,\mathrm{d}x \qquad \text{for } \zeta \to \infty \quad (21)$$

Proceeding along similar lines, we can also obtain $\hat{\phi}$ for large negative ζ

$$\hat{\phi}(\zeta,\omega,\lambda) \approx g\tilde{a}_1 \exp(-\mathrm{i}\tilde{l}_1\zeta)\int_{-\infty}^{\infty}\left(\hat{f} + U\frac{\partial \hat{P}}{\partial x} - \mathrm{i}(\omega - \lambda V)\hat{P}\right) \times \exp(\mathrm{i}\tilde{l}_1 x)\,\mathrm{d}x \qquad \text{for } \zeta \to -\infty \quad (22)$$

Setting Eq. (22) to zero gives

$$\int_{-\infty}^{\infty}\hat{f}\exp(\mathrm{i}\tilde{l}_1 x)\,\mathrm{d}x + \int_{-\infty}^{\infty}\left[U\frac{\partial \hat{P}}{\partial x} - \mathrm{i}(\omega - \lambda V)\hat{P}\right]\exp(\mathrm{i}\tilde{l}_1 x)\,\mathrm{d}x = 0 \quad (23)$$

Eq. (23) gives the required relationship between the two sources $\hat{f}(x)$ and $\hat{P}(x)$ to generate forward-propagating waves only.

To solve Eqs. (21) and (23) we need to know the form of $\hat{f}(x)$ and $\hat{P}(x)$. The choice is arbitrary and we choose a smooth Gaussian-shaped profile to obtain exact solutions for the integrals in Eqs. (21) and (23).

$$\hat{f}(x) = D_1 \exp(-\beta x^2) \quad (24a)$$

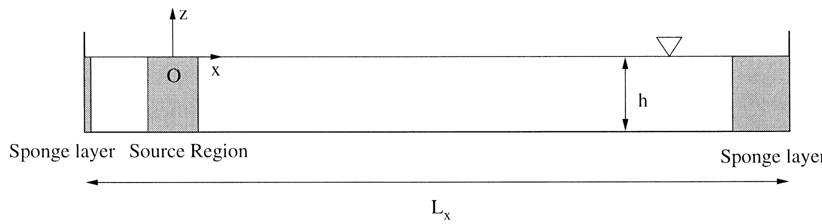

Fig. 2. Computational domain.

$$\hat{P}(x) = D_2 x \exp(-\beta x^2) \tag{24b}$$

where $D_1(\lambda, \omega, \beta)$, and $D_2(\lambda, \omega, \beta)$ are the unknown amplitudes of the source functions which have to be solved for. β is a free parameter related to the source width and is determined in the same way as in Ref. [11]. Defining the extent of the source as the roots of the equation

$$\exp[-\beta(x - x_s)^2] = \exp(-5) = 0.0067 \tag{25}$$

and defining the source width W in terms of a ratio of the wavelength L

$$W \equiv \gamma_r \frac{L}{2}$$

we can get an expression for β in terms of the wavelength.

$$\beta = \frac{80}{\gamma_r^2 L^2} \tag{26}$$

The free parameter is now γ_r instead of β. A smaller value of γ_r is preferred as it leads to a narrower source region. However, too small a value may lead to numerical problems resulting from finite differencing of the narrow region. Typical values of γ_r are around 0.5, leading to a source width on the order of a fourth of a wavelength. Fig. 1 shows the shape of the two sources as a function of x. We have chosen a symmetric shape for $\hat{f}(x)$ and an antisymmetric shape for $\hat{P}(x)$. The choice of a $\hat{P}(x)$ which breaks the symmetry of $\hat{f}(x)$ is needed because otherwise, in the absence of a current, Eq. (23) would cancel out wave motion both in front of and behind the source region.

Substituting Eq. (24) in Eq. (23) and solving gives

$$D_2 = \frac{-2\beta D_1}{\tilde{l}_1(\omega - \lambda V + \tilde{l}_1 U)} \tag{27}$$

To obtain a relation between the source function amplitude and wave motion amplitude, we Fourier transform the linear momentum equation (4b) in y and t

$$g\hat{\eta} = -\left[-\mathrm{i}(\omega - \lambda V) + U\frac{\partial}{\partial x}\right]\left[1 + \alpha h^2\left(\frac{\partial^2}{\partial x^2} - \lambda^2\right)\right]\hat{\phi} \tag{28}$$

Since we are considering progressive wave motion, we take

$$\hat{\eta}(x) = \eta_0 \exp(\mathrm{i} l_1 x)$$

where η_0 is the wave amplitude. Substituting for $\hat{\phi}$ from Eq. (21) and for the source functions from Eqs. (24) and (27), we get

$$D_1 = \frac{-\mathrm{i}\eta_0 \exp\left(\frac{l_1^2}{4\beta}\right)}{a_1\sqrt{\frac{\pi}{\beta}}\left[1 + \frac{l_1(\omega - \lambda V - l_1 U)}{\tilde{l}_1(\omega - \lambda V + \tilde{l}_1 U)}\right](\omega - \lambda V - l_1 U)(1 - \alpha h^2(l_1^2 + \lambda^2))} \tag{29}$$

where a_1 is obtained from Eq. (18).

Thus, with the help of Eqs. (29) and (27) we can obtain the amplitudes of the source functions in the mass and momentum equations such that they cancel all wave motion behind the source region, and add up in front of the source region to give the desired wave motion.

3. Numerical examples

We shall now test our modified source function model derived in the previous section with the help of the 1D version of the WKGS model. A schematic view of the test condition is shown in Fig. 2. The total length of the domain is $L_x = 30$ m and water depth $h = 0.5$ m. The center of the source is at $x_s = 0$ m. A fairly large sponge layer has been provided at the downwave end of the domain to absorb the wave energy.

We consider monochromatic waves with $T = 1.5$ s and $H = 0.02$ m. These are small waves and the aim is to see whether, in the linear limit, our source function method is able to reproduce the desired wave height. The grid size used in the model is $\Delta x = 0.025$ m and $\Delta t = 0.00458$ s.

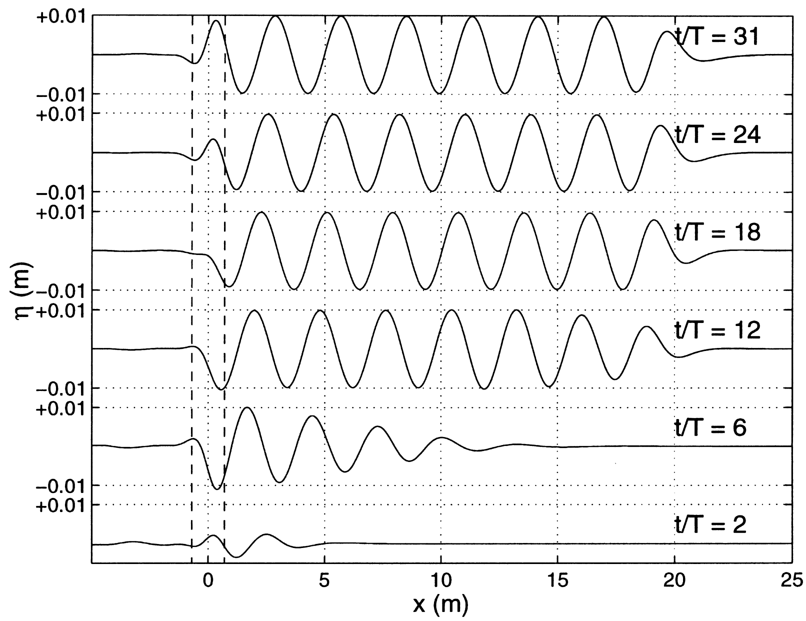

Fig. 3. Snapshot of surface elevation η, $T = 1.5$ s, $H = 0.02$ m, $Fr = 0$.

Fig. 3 shows snapshots of the surface elevation $\eta(x)$ at various times. The simulations have been done using the linearized version of the WKGS model and the dashed lines in the plot denote the extent of the source function. The two-source method works well in creating waves of a desired wave height along one direction only. There are no currents in this simulation.

While developing our model we had claimed that the boundary terms in Eq. (19), which do not cancel out in the presence of a current, are small and can be neglected. To test this claim we considered the case of the monochromatic wave moving over a strong opposing current (Fig. 4). Since we are considering a strong current, the wave–current interaction terms cannot be ignored and the simulations have to be conducted with the fully nonlinear version of the WKGS model. The desired wave is reproduced very well. Thus the boundary terms do turn out to have negligible effects. For the case of a strong following current (Fig. 5) the generated wave pattern is not perfectly sinusoidal. This is due to nonlinear triad interactions which are enhanced in the presence of following currents (Chen et al. [1]).

The source function method has been developed using linear wave theory and is expected to exhibit errors for large wave heights. To test the limits of this theory, a

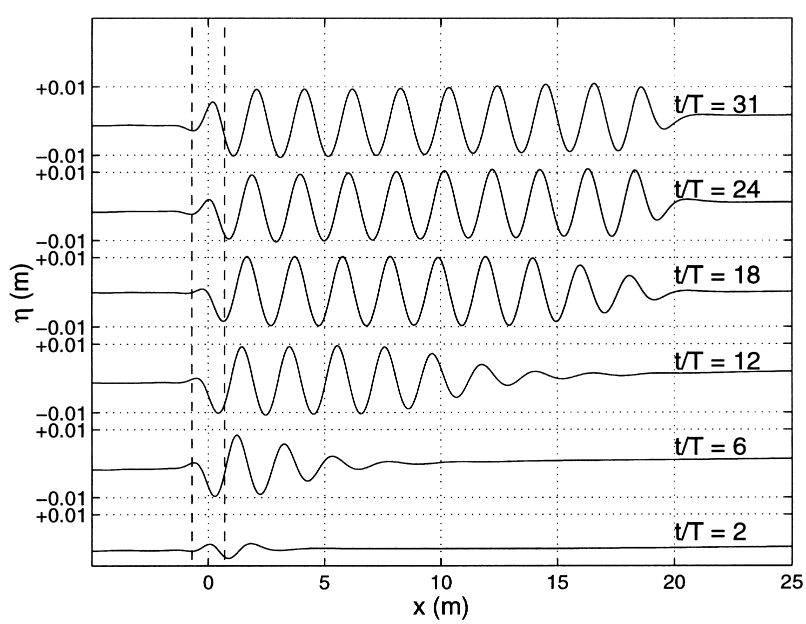

Fig. 4. Snapshot of surface elevation η, $T = 1.5$ s, $H = 0.02$ m, $Fr = -0.15$.

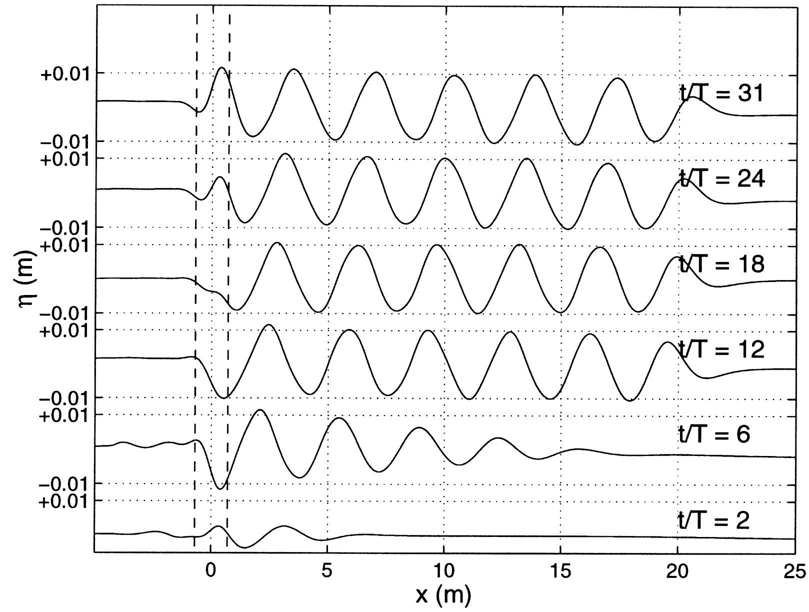

Fig. 5. Snapshot of surface elevation η, $T = 1.5$ s, $H = 0.02$ m, $Fr = 0.15$.

comparison between the measured wave height (obtained from the WKGS model at $x = 10$ m) and the desired wave height, for different values of δ is shown in Fig. 6. The error is very small ($\pm 2\%$) up to $\delta = 0.11$, and increases for larger values of δ due to nonlinear effects. Note that no attempt is made to directly generate higher harmonics in this simulation.

The present one-way source function method is a modification of the two-way method of Wei et al. [11]. Further tests on a distributed source function method are given in that paper and the reader should refer to it for further examples of this method for generating waves.

4. Conclusions

In this paper, we have described an alternate formulation for generating waves in weakly dispersive wave models using two spatially distributed source functions. The formulation allows for the existence of an underlying O(1) current field, and thus can be directly utilized in studies involving interaction of waves with a strong current. The method was tested with the WKGS model both in the presence and absence of strong currents and found to work well. Even though the method uses the linear wave approximation it reproduces the larger wave heights with reasonable accuracy.

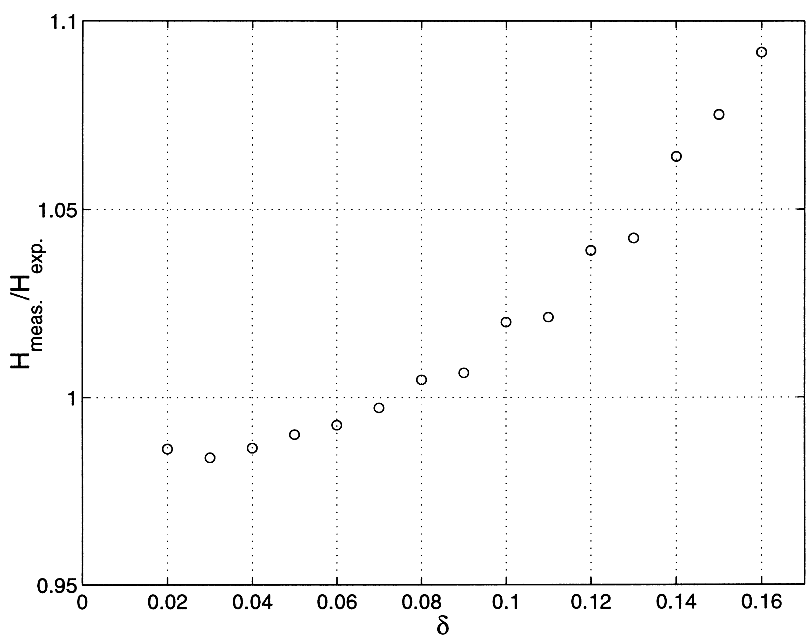

Fig. 6. Comparison between the computed and expected wave height for different values of $\delta = H_{\mathrm{exp}}/2h$ ($T = 1.5$ s, $Fr = -0.15$).

This present formulation generates waves traveling in one direction only, as opposed to the method used by Wei et al. [11] and GK99. The advantage of this method is that it eliminates non-physical backward propagating waves, thus greatly reducing the required damping layer behind the source region in cases where reflection from the modeled domain is small.

An alternative technique for generating waves along one direction, developed by Skotner and Apelt [9], was brought to the attention of the authors during the review process of the present paper. They use a single generation point (line in a 2D model) as opposed to a finite source region, and instead of using two sources they modify the spatial derivatives on the incident and reflected sides of the generation point. The advantage of their indirect method is that they do not require a finite source region and can further reduce the domain size. However, this is done at the expense of a number of additional arrays needed to store information on both sides of the generation line. The method described here is straightforward to implement and does not modify the numerical model, except for the addition of an extra term in the continuity and momentum equations. It is not immediately clear which of the two methods would be more computationally efficient, and a more detailed comparison of the two methods would be required.

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Appendix A. Theory for the O(kh)^4 model of GK99

The set of governing equations in non-dimensional form for the linear version of the O(kh)^4 model of GK99 is given by

$$\frac{d\eta}{dt} + h\nabla\cdot\mathbf{u}_w + \mu^2\frac{h^3}{2}(\gamma_1 - 1/3)\nabla^2(\nabla\cdot\mathbf{u}_w) + \mu^4\frac{h^5}{4}\left[\gamma_1(\gamma_1 - 1/3) - \frac{(\gamma_2 - 1/5)}{6}\right]\nabla^2(\nabla^2(\nabla\cdot\mathbf{u}_w)) = 0 \tag{A1a}$$

$$\frac{d\mathbf{u}_w}{dt} + \nabla\eta + \mu^2\frac{h^2}{2}(\gamma_1 - 1)\frac{d}{dt}\{\nabla(\nabla\cdot\mathbf{u}_w)\} + \mu^4\frac{h^4}{4}\left[\gamma_1(\gamma_1 - 1) - \frac{(\gamma_2 - 1)}{6}\right]\frac{d}{dt}\{\nabla(\nabla^2(\nabla\cdot\mathbf{u}_w))\} = 0 \tag{A1b}$$

where γ_1 and γ_2 are given by

$$\gamma_1 \equiv \frac{1}{h^2}[\beta(h + z_a)^2 + (1 - \beta)(h + z_b)^2] \tag{A2a}$$

$$\gamma_2 \equiv \frac{1}{h^4}[\beta(h + z_a)^4 + (1 - \beta)(h + z_b)^4] \tag{A2b}$$

and $\mathbf{u}_w$ is defined by weighting the velocities at the reference depths z_a and z_b

$$\mathbf{u}_w = \beta\mathbf{u}_a + (1 - \beta)\mathbf{u}_b \tag{A3}$$

β, z_a and z_b are chosen to obtain appropriate dispersion characteristics even in deeper waters (see GK99).

Writing the equations in dimensional form, using a potential function $\phi(x, y, t)$ and introducing source functions we get a combined equation of the form

$$\frac{d^2\phi}{dt^2} - gh\nabla^2\phi - g\frac{h^3}{2}C_1\nabla^2\nabla^2\phi - g\frac{h^5}{4}C_2\nabla^2\nabla^2\nabla^2\phi + \frac{h^2}{2}C_3\nabla^2\left(\frac{d^2\phi}{dt^2}\right) + \frac{h^4}{4}C_4\nabla^2\nabla^2\left(\frac{d^2\phi}{dt^2}\right) = -g\left(f + \frac{dP}{dt}\right) \tag{A4}$$

where $C_1 \equiv \gamma_1 - 1/3$, $C_2 \equiv \gamma_1(\gamma_1 - 1/3) - ((\gamma_2 - 1/5)/6)$, $C_3 \equiv \gamma_1 - 1$ and $C_4 \equiv \gamma_1(\gamma_1 - 1) - ((\gamma_2 - 1)/6)$.

Taking a Fourier transform along both y and t yields the following ordinary differential equation

$$A\hat{\phi}^{(6)} + B\hat{\phi}^{(5)} + C\hat{\phi}^{(4)} + D\hat{\phi}^{(3)} + E\hat{\phi}^{(2)} + F\hat{\phi}^{(1)} + K\hat{\phi} = g(\hat{f} + U\hat{P}^{(1)} - \mathrm{i}(\omega - \lambda V)\hat{P}) \tag{A5}$$

where

$$A \equiv C_2 g\frac{h^5}{4} - C_4U^2\frac{h^4}{4} \tag{A6a}$$

$$B \equiv 2\mathrm{i}(\omega - \lambda V)UC_4\frac{h^4}{4} \tag{A6b}$$

$$C \equiv g\frac{h^3}{2}\left(C_1 - \frac{3}{2}C_2\lambda^2h^2\right) - C_3U^2\frac{h^2}{2} + C_4\frac{h^4}{4}((\omega - \lambda V)^2 + 2\lambda^2U^2) \tag{A6c}$$

$$D \equiv \mathrm{i}(\omega - \lambda V)Uh^2(C_3 - \lambda^2h^2C_4) \tag{A6d}$$

$$E \equiv gh - U^2 - \lambda^2gh^3\left(C_1 - \frac{3}{4}\lambda^2h^2C_2\right) + C_3\frac{h^2}{2}((\omega - \lambda V)^2 + \lambda^2U^2) - C_4\frac{h^4}{4}\lambda^2(2(\omega - \lambda V)^2 + \lambda^2U^2) \tag{A6e}$$

$$F \equiv 2\mathrm{i}(\omega - \lambda V)U(1 - \lambda^2 + \lambda^4) \tag{A6f}$$

$$K \equiv (\omega - \lambda V)^2 - gh\lambda^2 + \lambda^4 g \frac{h^3}{2}\left(C_1 + \lambda^2 \frac{h^2}{2} C_2\right) - \lambda^2(\omega - \lambda V)^2 \frac{h^2}{2}\left(C_3 - C_4\lambda^2 \frac{h^2}{2}\right) \tag{A6g}$$

Once again using the method of Green's function and following the steps given before, we obtain

$$G(\zeta, x) = \tilde{a}_1 \exp[\mathrm{i}\tilde{l}_1(x - \zeta)] + \tilde{a}_2 \exp[\mathrm{i}\tilde{l}_2(x - \zeta)] + \tilde{a}_3 \exp[\mathrm{i}\tilde{l}_3(x - \zeta)] \tag{A7a}$$

for $x > \zeta$

$$G(\zeta, x) = a_1 \exp[\mathrm{i}\tilde{l}_1(\zeta - x)] + a_2 \exp[\mathrm{i}\tilde{l}_2(\zeta - x)] + a_3 \exp[i\tilde{l}_3(\zeta - x)] \tag{A7b}$$

for $x < \zeta$

where $\tilde{l}_1$ and l_1 are real and $\tilde{l}_2$, $\tilde{l}_3$, l_2, l_3 are complex. The coefficients are obtained from a set of matching conditions similar to the $O(kh)^2$ solution and are given by

$$\tilde{a}_1 = \frac{\mathrm{i}}{A(\tilde{l}_1 - \tilde{l}_3)(\tilde{l}_2 - \tilde{l}_1)(\tilde{l}_1 + l_1)(\tilde{l}_1 + l_2)(\tilde{l}_1 + l_3)} \tag{A8a}$$

$$\tilde{a}_2 = \frac{\mathrm{i}}{A(\tilde{l}_2 - \tilde{l}_1)(\tilde{l}_3 - \tilde{l}_2)(\tilde{l}_2 + l_1)(\tilde{l}_2 + l_2)(\tilde{l}_2 + l_3)} \tag{A8b}$$

$$\tilde{a}_3 = \frac{\mathrm{i}}{A(\tilde{l}_3 - \tilde{l}_1)(\tilde{l}_2 - \tilde{l}_3)(\tilde{l}_3 + l_1)(\tilde{l}_3 + l_2)(\tilde{l}_3 + l_3)} \tag{A8c}$$

$$a_1 = \frac{\mathrm{i}}{A(l_1 - l_2)(l_3 - l_1)(l_1 + \tilde{l}_1)(l_1 + \tilde{l}_2)(l_1 + \tilde{l}_3)} \tag{A8d}$$

$$a_2 = \frac{\mathrm{i}}{A(l_2 - l_3)(l_1 - l_2)(l_2 + \tilde{l}_1)(l_2 + \tilde{l}_2)(l_2 + \tilde{l}_3)} \tag{A8e}$$

$$a_3 = \frac{\mathrm{i}}{A(l_3 - l_1)(l_2 - l_3)(l_3 + \tilde{l}_1)(l_3 + \tilde{l}_2)(l_3 + \tilde{l}_3)} \tag{A8f}$$

From here on the solution follows the $O(kh)^2$ solution very closely, and thus, the intermediate steps have been omitted. The final relationship between the wave amplitude η_0 and source function amplitude D_1 is given by

$$D_1 = \frac{-\mathrm{i}\eta_0 \exp\left(\dfrac{l_1^2}{4\beta}\right)}{a_1\sqrt{\dfrac{\pi}{\beta}}\left[1 + \dfrac{l_1(\omega - \lambda V - l_1 U)}{\tilde{l}_1(\omega - \lambda V + \tilde{l}_1 U)}\right](\omega - \lambda V - l_1 U)\left[1 - \dfrac{h^2}{2}(l_1^2 + \lambda^2)\left(C_3 - \dfrac{h^2}{2}C_4(l_1^2 + \lambda^2)\right)\right]} \tag{A9}$$

with Eq. (27) remaining unchanged.

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