EXPERIMENTAL STUDY OF BREAKING WAVES OVER A SHOAL

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ABSTRACT: The aim of this paper is to study the transformation of irregular directional waves over a circular shoal. An experimental study has been carried out. The resulting data has been used to test the accuracy of an existing refraction-diffraction model. Model to data comparisons have been carried out for the entire basin region including about 3 shoal diameters downwave of the shoal, with satisfactory results. Several physical processes have been identified which lead to possible disparities in the comparisons.

INTRODUCTION

Wave modeling of random waves over a varying bathymetry is a subject of considerable importance to coastal engineers. The development of the classical mild slope equation by Berkhoff (1972) allowed coastal engineers to study the combined effects of refraction and diffraction. A wide family of equations have been derived from the mild slope equation to increase the accuracy and the speed of the models. One such set of equations are the parabolic equations, first derived for ocean waves by Radder (1979), which have gained popularity because of their speed of computation, even though they have a fixed direction of propagation and a limited range of angles from the assumed propagation direction over which they are valid. Nonlinear formulations of parabolic equations (Kirby and Dalrymple 1983) have been found to give more accurate results than the linear mild slope equation (Kirby and Dalrymple 1984). The limitation in the range of angles has been relaxed using Padé approximants (Booij, 1981).

Although relatively accurate parabolic models have been developed to study the evolution of waves over an irregular bottom, all these models have been derived for monochromatic waves only. Coastal engineers have traditionally approximated the offshore irregular sea states by representative monochromatic waves in order to use these models to make predictions. However, investigators such as Goda (1985) (using an analytical approach), Vincent and Briggs (1989) (by conducting an experimental study) and Panchang *et al.* (1990) (using a numerical approach) have shown that such an approximation may result in large errors due to vast dissimilarities in the refraction-diffraction patterns of the two

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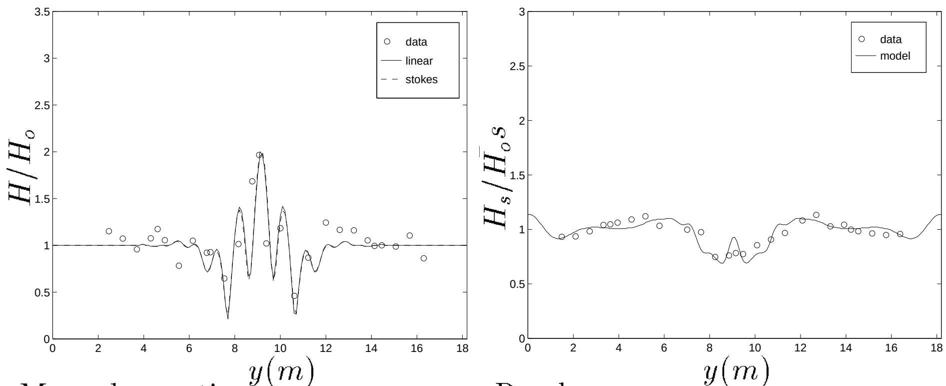


Monochromatic waves Random waves

Figure 1: Wave height distribution behind a shoal for a monochromatic wave and a directional sea state

wave fields. Our own experiments confirm this and Figure 1 shows the vast differences in the wave height distribution along a transect (transect D-D in Figure 2) behind a submerged shoal.

Recently, methods for computing the evolution characteristics of a directional spectral sea state using parabolic models for monochromatic waves have been developed. Panchang *et al.* (1990), Grassa (1990) and Izumiya and Horikawa (1987) have developed models using a spectral calculation method which consists of discretizing the offshore spectrum into individual monochromatic directional components, determining the wave transformations of each component with the help of monochromatic wave models, and then assembling the wave components by linear superposition at the respective grid points in the domain to obtain the statistical characteristics of the spectrum at those points.

In this paper a numerical model which has been developed using the parabolic formulation of Kirby (1986a), is tested against data for a range of breaking wave conditions. Experimental study of random directional waves breaking over a submerged circular shoal has been carried out for two different directional spreadings and energy variances to study their effects on wave height distribution. Extensive surface elevation measurements have been made on top of and around the shoal, and some aspects of the frequency spectra have been looked into.

NUMERICAL MODEL

The parabolic model for spectral wave conditions used here simulates the evolution of directional random waves in the nearshore zone. The model predicts the effects of refraction, diffraction, shoaling and breaking. Therefore, the model is particularly applicable to regions where an incoming random sea propagates over complicated bathymetry towards shore. The bathymetry may include a shoal formation at the mouth of an inlet or estuary, where refraction, diffraction, shoaling and depth-limited breaking will be simultaneously important.

The model requires the specification of the incoming directional random sea at the offshore boundary. The random sea is represented by a two-dimensional spectrum which is discretized into wave components, resulting in wave components of amplitude A with associated frequency f and angle of incidence θ to

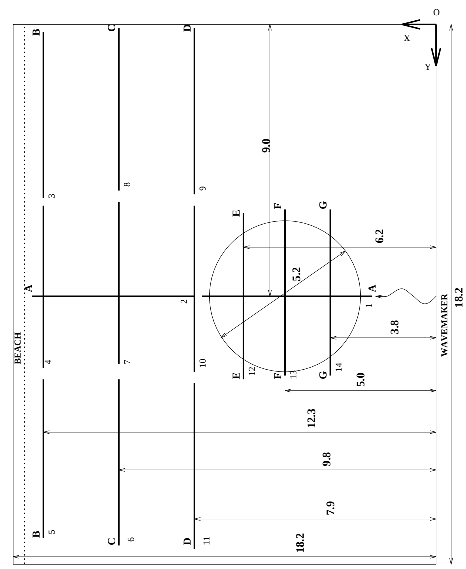


Figure 2: Schematic diagram of the experimental setup

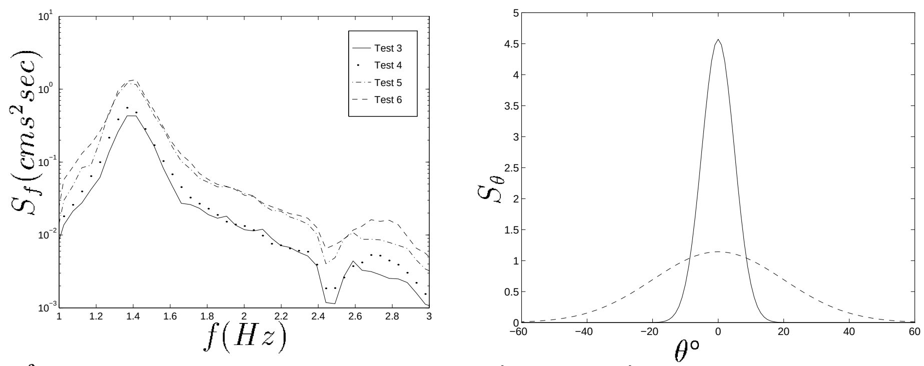
the assumed propagation direction, herein called x direction. The water surface elevation can be described in terms of these discrete wave components. It is assumed that the water surface elevation η is periodic in time and that the spatial dependency can be split into a fast-varying phase and a slow-varying amplitude.

$$\eta(x,y,t) = \sum_{all f} \sum_{all \theta} \left\{ \frac{A(x,y;f,\theta)}{2} e^{i\psi} + c.c. \right\} \tag{1}$$

where f is the frequency, θ is the direction of any individual wave component and

$$\psi = \int \mathbf{k} \cdot d\mathbf{x} - \omega t \tag{2}$$

The evolution of these individual wave components is computed simultaneously at each forward step in the assumed wave propagation direction using a monochromatic wave model. Therefore, after each forward step it is possible to determine statistical quantities at that row before taking another step forward. These quantities are incorporated into a statistical wave breaking model (Thornton and Guza, 1983) which has been added to the monochromatic wave model. The refraction, diffraction and shoaling of the discrete wave components is assumed to be governed by the parabolic approximation to the mild slope equation derived by Berkhoff (1972). To minimize the restrictions placed on the range of allowed wave angles with respect to the assumed wave direction, the procedure derived by Booij (1981) is used, enabling the model to handle wave direction up to about 45° from the x direction. The model also has the ability to handle strong currents by using the formulation of the mild slope equation including the influence of currents derived by Kirby (1986a). The governing equation for the wave component n is:



frequency spectra | directional spectra

Figure 3: Design frequency and directional spectra for the 4 directional sea states

Table 1: Test particulars for the random wave experiments

Test no.	$H_{0s}(m)$	$T_p(sec)$	θ_m	Range
3	0.0139	0.73	0	±15°
4	0.0156	0.73	0	±45°
5	0.0233	0.73	0	±15°
6	0.0249	0.71	0	±45°

$$
\begin{aligned}
& (C_{gn}+U)(A_n)_x - 2\Delta_1 V(A_n)_y + i(\bar{k}_n - a_0 k_n)(C_{gn}+U)A_n \\
+ & \left\{\frac{\sigma_n}{2}\left(\frac{C_{gn}+U}{\sigma_n}\right)_x - \Delta_1\sigma_n\left(\frac{V}{\sigma_n}\right)_y\right\}A_n + i\Delta_n'\left[\left((CC_g)_n - V^2\right)\left(\frac{A_n}{\sigma_n}\right)_y\right]_y \\
- & i\Delta_1\left\{\left[UV\left(\frac{A_n}{\sigma_n}\right)_y\right]_x + \left[UV\left(\frac{A_n}{\sigma_n}\right)_x\right]_y\right\} + \alpha A_n \\
+ & \frac{-b_1}{k_n}\left\{\left[\left((CC_g)_n - V^2\right)\left(\frac{A_n}{\sigma_n}\right)_y\right]_{yx} + 2i\left(\sigma_n V\left(\frac{A_n}{\sigma_n}\right)_y\right)_x\right\} \\
+ & b_1\beta_n\left\{2i\omega_n U\left(\frac{A_n}{\sigma_n}\right)_x + 2i\sigma_n V\left(\frac{A_n}{\sigma_n}\right)_y - 2UV\left(\frac{A_n}{\sigma_n}\right)_{xy}\right. \\
+ & \left.\left[\left((CC_g)_n - V^2\right)\left(\frac{A_n}{\sigma_n}\right)_y\right]_y\right\} - \frac{i}{k_n}b_1\left\{(\omega_n V)_y + 3(\omega_n U)_x\right\}\left(\frac{A_n}{\sigma_n}\right)_x \\
- & \Delta_2\left\{\omega_n U\left(\frac{A_n}{\sigma_n}\right)_x + \frac{1}{2}\omega_n U_x\left(\frac{A_n}{\sigma_n}\right)\right\} + ik_n\omega_n U(a_0-1)\left(\frac{A_n}{\sigma_n}\right) = 0 \qquad (3)
\end{aligned}
$$

where U and V are the currents in the x and y directions, α is a dissipation coefficient for wave breaking, $\overline{k}_n$ is a representative wave number corresponding

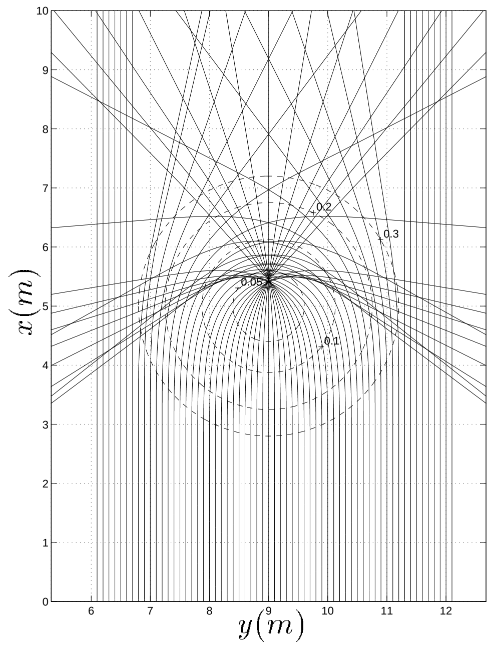


Figure 4: Refraction diagram at peak frequency, $f_p = 1.37Hz$ for the given bathymetry.

to the peak frequency, and

$$
\begin{aligned}
\sigma_n &= \omega_n - k_n U, \\
\beta_n &= \frac{(k_n)_x}{k_n^2} + \frac{(k_n\left((CC_g)_n - U^2\right))_x}{2k_n^2((CC_g)_n - U^2)}, \\
\Delta_1 &= a_1 - b_1, \\
\Delta_2 &= 1 + 2a_1 - 2b_1, \\
\Delta_n' &= a_1 - b_1 \frac{\overline{k}_n}{k_n}.
\end{aligned}
\qquad (4)
$$

The model coefficients

$$a_0 = 1; \qquad a_1 = -0.75; \qquad b_1 = -0.25 \qquad (5)$$

recover the Padé approximant of Booij(1981).

The statistical information obtained after each step in the parabolic scheme is used to construct a model for the dissipation of energy due to breaking.To determine the energy dissipation, a simple model by Thornton and Guza (1983) is used. The energy dissipation model is built into the model equation using an additional breaking term αA_n in (3), so that it is unnecessary to have any criterion for turning breaking on or off. The coefficient α is given by

$$\alpha = \frac{3\sqrt{\pi}}{4} \frac{\bar{f} B^3}{\gamma^4 h^5} H_{rms}^5. \qquad (6)$$

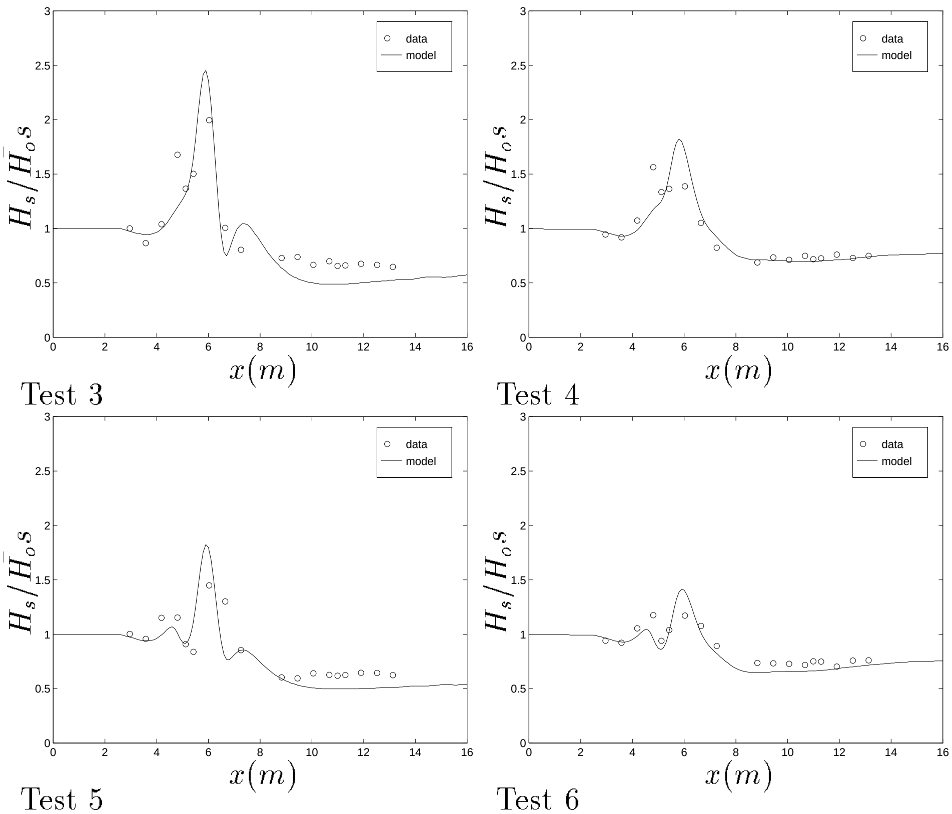


Figure 5: Significant wave height distribution along transect A-A

where h is the local water depth and $\bar{f}$ is a representative frequency for the frequency spectrum and is chosen to be the peak frequency. B and γ are constants and are chosen to be equal to 1 and 0.6, respectively (Mase and Kirby, 1992). H_{rms} is the root-mean-square wave height, and is obtained as a statistical quantity from the wave model,

$$H_{rms}(x,y) = 2\sqrt{\sum_{n=1}^{N} |\, A(x,y)_n \,|^2} \tag{7}$$

The dissipation model of Thornton and Guza (1983) was originally derived assuming that the waves continue breaking once they have started, and has been validated for waves breaking on a monotonic beach. Though we can see from (6) that the dissipation term α is artificially reduced with increase in local water depth h we are not certain if this correctly simulates the reforming of waves with increased water depth. Also, no modifications have been made in the dissipation model to account for directional effects. Only change in energy flux in the x direction is considered, and energy flux in the y direction does not take part in the dissipation model.

EXPERIMENTAL SETUP

The experiments were conducted at the Center for Applied Coastal Research, University of Delaware. The wave basin is approximately 18.2m long and 18.2m

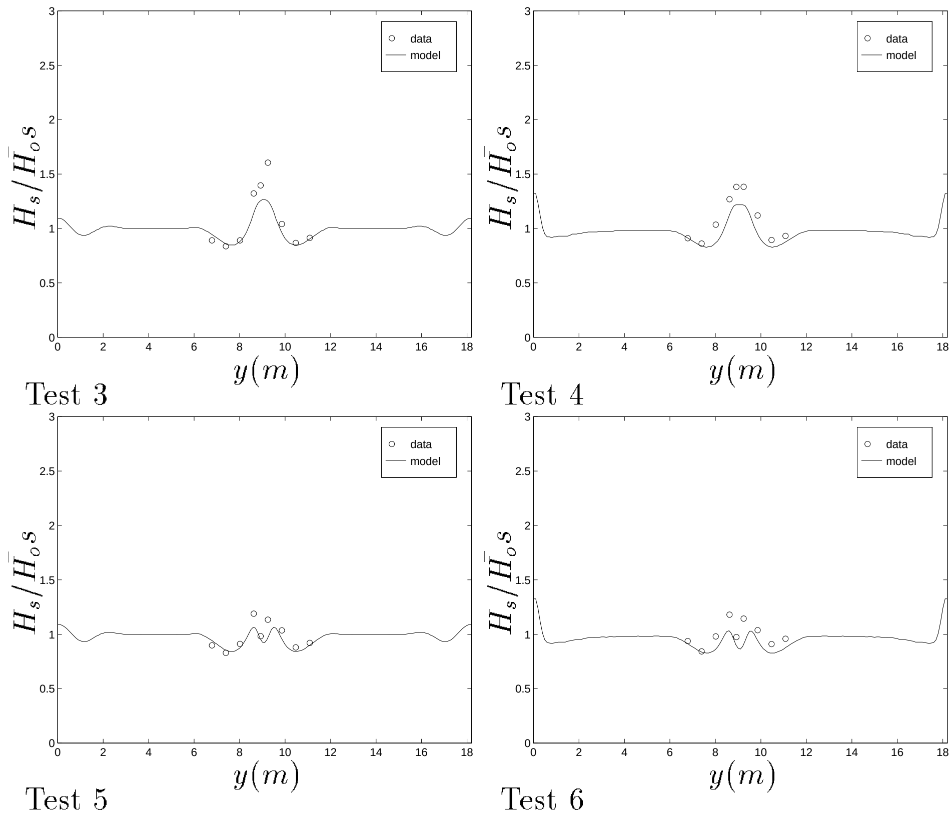


Figure 6: Significant wave height distribution along transect F-F

wide. It has a three-dimensional wavemaker at one end, consisting of 34 flap type paddles which creates the desired wave field. The bottom is flat except for a circular shoal in the center, and a stone beach at the far end minimizes the reflections. A schematic view of the experimental layout, together with the gage locations is given in Figure 2.

A total of ten capacitance wave gages were used in the experiment, of which nine were placed on an array. This array was then placed at fourteen different positions (denoted by thick lines in Figure 2) to obtain a total of 126 measuring points around the shoal. Depending upon their orientation, one or more array positions form a transect along which comparisons are made with the numerical model. There is one longitudinal transect (A-A) going over the top of the shoal and six transverse transects (B-B, C-C, D-D, E-E, F-F and G-G) behind and on top of the shoal (see Figure 2).

The circular shoal has a diameter of $5.2m$ and a maximum height of $37cm$. Geometrically it is the top portion of a circular sphere of radius $9.1m$. The center of the shoal is placed at $x = 5m$ and $y = 8.98m$. The equation for the perimeter of the shoal is given by

$$(x-5)^2 + (y-8.98)^2 = (2.57)^2 \tag{8}$$

and for the bathymetry is given by

$$z = -h + \sqrt{82.81 - (x-5)^2 - (y-8.98)^2} - 8.73 \tag{9}$$

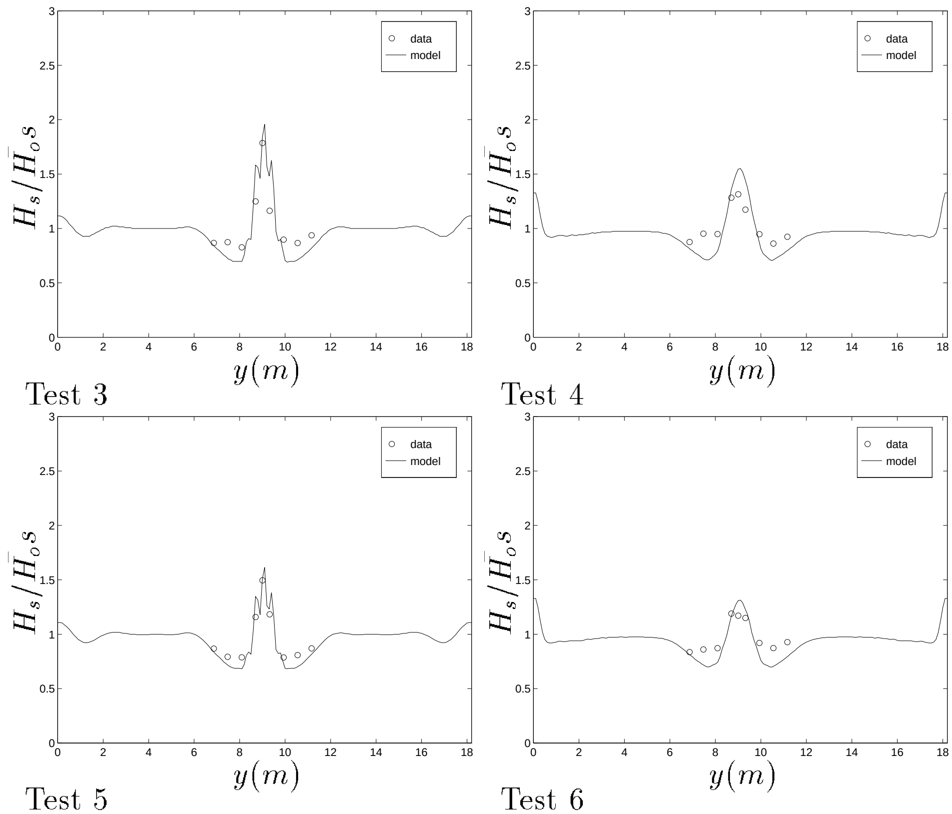


Figure 7: Significant wave height distribution along transect E-E

where h is the water depth away from the shoal.

Four different directional sea test conditions (Test 3, Test 4, Test 5 and Test 6) were run with a TMA spreading function (Bouws *et al.* , 1985) in frequency, and a wrapped normal directional spreading function (Borgman, 1984) in direction. The water depth away from the shoal (h in (9)) was $40cm$, and the water depth on top of the shoal was $3cm$. All four tests had similar frequency spreadings except that the energy variance in Tests 3 and 4 were lower, and the frequency spectra for the four test cases is given in Figure 3. In all the four cases the waves were breaking on top of the shoal, with more waves breaking for Tests 5 and 6. Two different directional spreadings were used (Figure 3), with the mean angle normal to the wavemaker ($\theta_m = 0°$). Tests 3 and 5 have a narrow directional spread ($\pm 11°$), while Tests 4 and 6 have a broad directional spread ($\pm 45°$).

The initial significant wave height (H_{0s}), peak period (T_p), mean angle (θ_m) and the range of directional spreading for the four different test cases are given in Table 1. Data was collected at a sampling rate of $50Hz$ for 655 seconds (32768 sample points) at all the gages.

DATA TO MODEL COMPARISONS

Significant wave height information was obtained from the data using a zero-upcrossing method, while from the model it was obtained from the statistics assuming a Rayleigh wave height distribution. Reflections from the beach at the

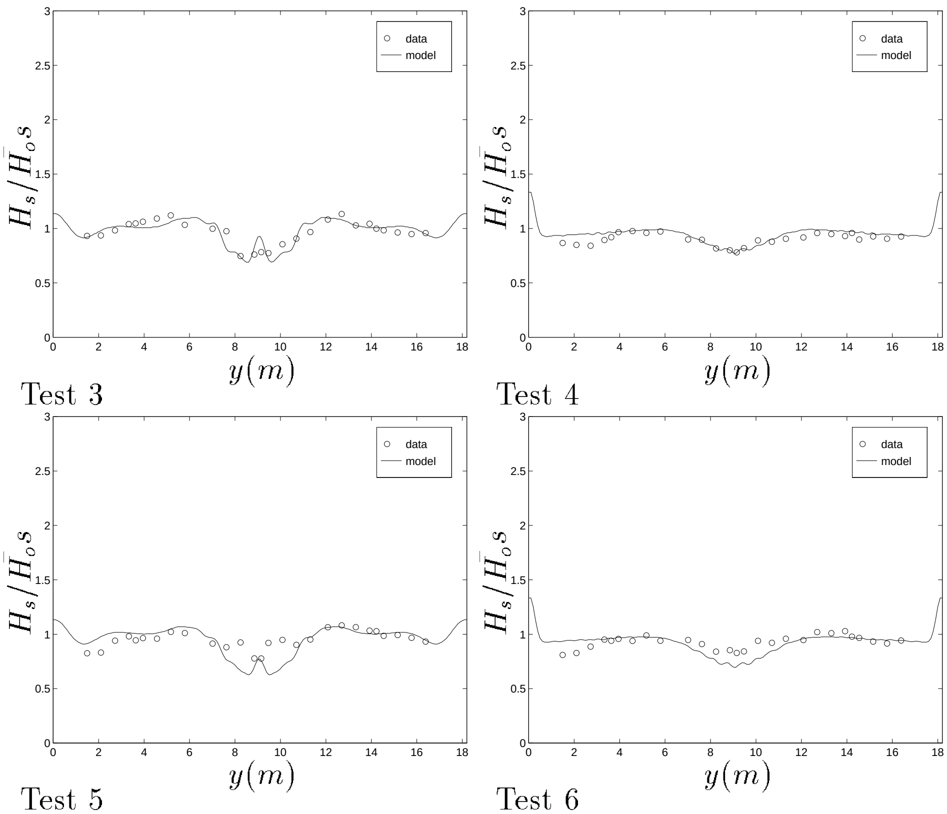


Figure 8: Significant wave height distribution along transect D-D

far end of the basin (see Figure 2) are a matter of concern but have been ignored here since the reflected wave field could not be separated from the incident wave field. In each case the input frequency spectrum to the model was directly measured from the wave data. The input directional spectrum was taken to be the same directional spreading function used to generate incident waves for the respective test cases (Figure 3).

A wave refraction pattern for the peak frequency (Figure 4) shows that the focusing is quite severe on top of the shoal, and that some of the wave rays are moving at angles greater than 90°. Since the model can predict wave conditions accurately only within a range of wave angles of ±45°, some discrepancies between model and data results are expected in this region.

For each test case, all significant wave height comparisons have been normalized by the respective initial significant wave height (H_{0s}) given in Table 1. Figure 5 gives the wave height comparisons along transect A-A. The model tends to overpredict the wave height distribution near the region of focus. This is probably due to the severe focusing in this region (Figure 4), which the model cannot properly simulate. Since the focusing is taking place inside the surf zone, another probable cause for the discrepancy could be the limitations of the breaking model, and a different breaking model might give more accurate results.

Comparisons along the six transverse transects are shown in Figures $6-10$. In all the cases the model predicts large wave heights at the side walls. This is because the no flux boundary condition at the side wall causes the waves to

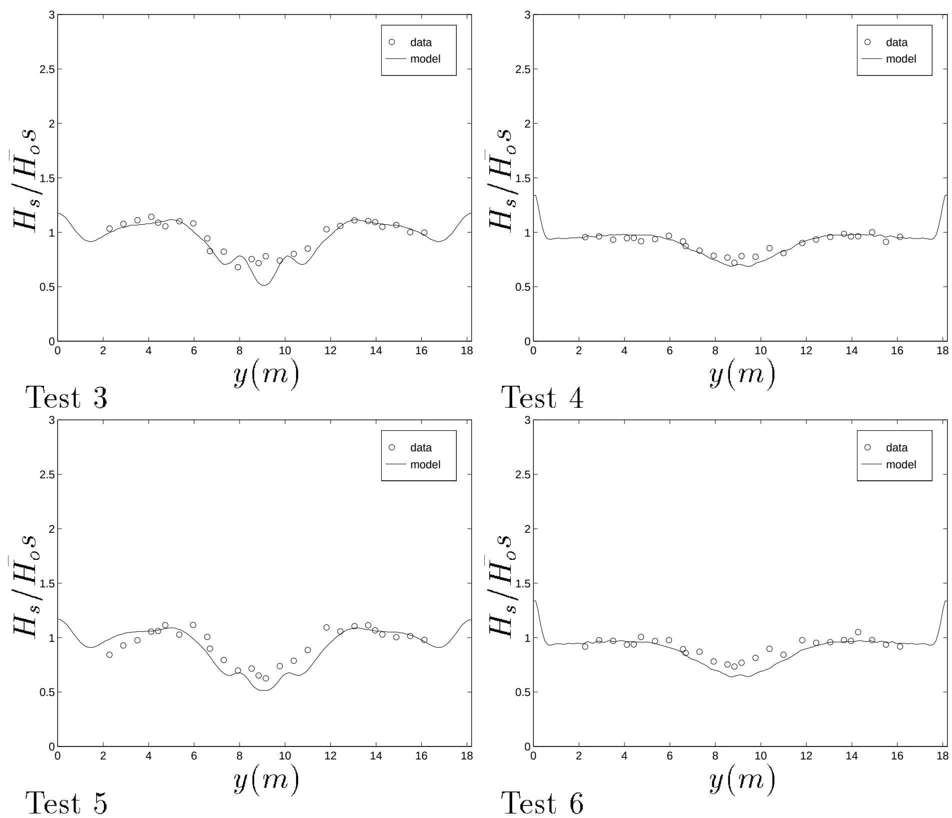


Figure 9: Significant wave height distribution along transect C-C

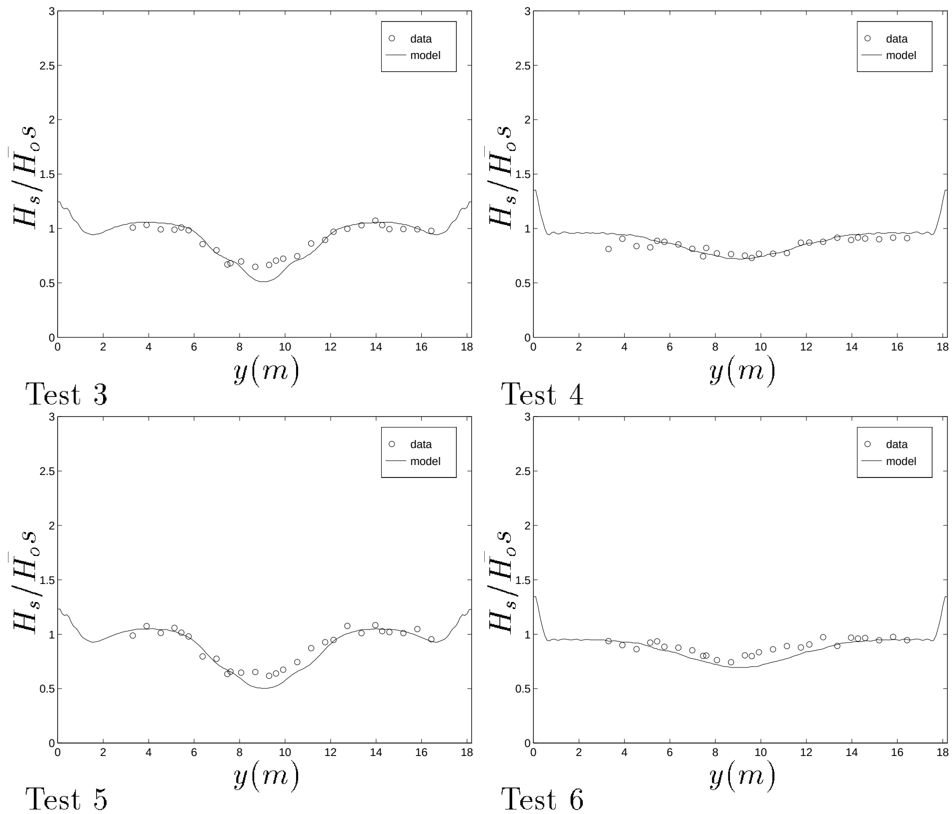
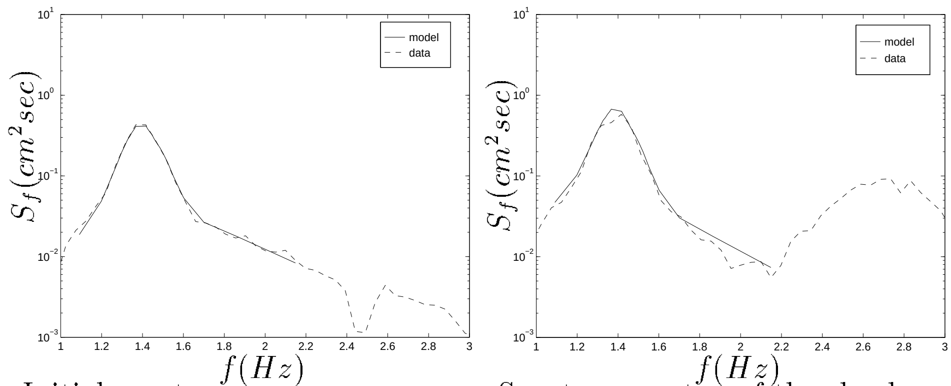


Figure 10: Significant wave height distribution along transect B-B

form an antinode there for each wave component, which when superimposed lead to large significant wave heights. On top of the shoal (Figure 6) and along transect E-E (Figure 7) where the waves are breaking and focusing, the same energy discrepancy that was seen in Figure 5 is observed, but the spread of the wave heights is simulated reasonably well by the model. The comparisons further behind the shoal (Figures 8 − 10) on the other hand are extremely good. In general we see that the wave height distribution behind the shoal is more smoothed out for the broad directional test cases (Tests 4 and 6) as compared to the narrow directional test cases (Tests 3 and 5). An interesting observation is that behind the shoal the wave height distributions are more a function of the type of directional distribution of the input spectrum, instead of being a function of the energy content of the spectrum. Before the focusing takes place (Figure 6), the wave height distributions for Tests 3 and 4, and Tests 5 and 6 are quite similar, while after focusing (Figures 6 − 10) Tests 3 and 5, and Tests 4 and 6 have similar wave height spreadings. This effect can also be seen clearly in Figure 5, where the wave height distribution till $x = 5m$ is a function of the energy content of the spectrum, and beyond that depends on the directional spreading of the spectrum.

Though the model gives reasonable significant wave height comparisons, it is unable to predict wave-wave interactions since it is based on a linear superposition of monochromatic wave components. These interactions lead to the formation of higher harmonics in nature, and become more pronounced with increased nonlinearity. A comparison of model spectra to data spectra on top of



Initial spectrum Spectrum on top of the shoal

Figure 11: Frequency spectra comparisons (Test 3) showing nonlinear wave-wave interactions on top of the shoal

the shoal (Figure 11) shows this disparity quite clearly. The higher harmonics (second peak) in the data have considerable amount of energy compared to the primary wave field (first peak), all of which are not predicted by the model. These higher harmonics are seen in the data on top of the shoal and in the region of focus where the wave field is highly nonlinear.

CONCLUSIONS

A parabolic model for simulating the evolution of wave spectra over a gently sloping bottom has been tested with experimental data. Wave height comparisons have shown that the model works reasonably well in simulating wave height distributions for breaking random waves. Some discrepancies exist in the region of focus which could be due to the parabolic limitations of the model. Discrepancies could also be due to limitations of the breaking model, and in predicting non-linear effects. To get a better idea as to whether the discrepancy between the data and the model on top of the shoal is due to the limitations of the numerical model, or errors in the experimental data, comparisons need to be made to a model which will be able to simulate waves with no limitations on the range of angles and also predict the generation of higher harmonics on top of the shoal.

Nonetheless, we find that the spectral model works well in simulating transformations of a random wave field over an irregular bathymetry, even for broad directional spectra. Although certain aspects of the frequency spectrum cannot be obtained accurately from the model, it can be used to obtain useful estimates of significant wave heights.

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