The Landslide-HySEA model

The Landslide-HySEA model is the first model of the HySEA family dealing with landslide generated tsunamis produced by a granular slide material. Currently we are working in other versions of this model including dispersion (to be used in BP#4). We are also working on implementing new models as a non-hydrostatic multilayer shallow-water system to be used for reproducing BP#2 and BP#4. Most of the work is currently in progress.

The Landslide-HySEA tsunami model implements the natural 2D extension of the 1D two-layer Savage-Hutter model presented in Fernández-Nieto et al. (2008), where Cartesian coordinates are used instead of local coordinates at each point of the 2D domain and where no anisotropy effects are taken into account in the normal stress tensor of the solid phase. The mathematical model consists of two systems of equations that are coupled: the model for the slide material is represented by a Savage-Hutter type of model (Savage and Hutter, 1989), and the water dynamics model is represented by the shallow-water equations (see Fernández-Nieto et al., 2008). One of the most important features of the model is that both the dynamics of the sedimentary fluidized material and the seawater layer are coupled and each of these two phases influences the other one instantly and they are computed simultaneously. These coupled effects were first studied in a 1D model by Jiang and Leblond (1992), who concluded that these effects are significant for cases of smaller slide material density and shallower waters. Nevertheless, the importance of numerically treating in a coupled mode phenomena that are physically coupled has been studied, for example, in Castro et al. (2011a) for the case of two-layer shallow-water fluids, and in Cordier et al. (2011) for sediment transport models. An uncoupled numerical treatment of these systems may generate spurious oscillations at the water surface or the interface.

The mathematical model implemented in the Landslide-HySEA tsunami code consists of a stratified media of two layers: the first layer is composed of a homogeneous inviscid fluid with constant density ρ_1 (sea water here), and the second layer represents the fluidized granular material with density ρ_s and porosity ψ_0. We assume that the mean density of the fluidized debris is constant and equals rho$_2 = (1 - \psi_0)\, \rho_s + \psi_0\, \rho_1$ and that the two fluids (water and fluidized debris) are immiscible.

The resulting system of equations writes as follows:

$$(h_1)_t+(q_{1,x})_x+(q_{1,y})_y = 0$$

$$(q_{1,x})_t+(q_{1,x}^2/h_1 + g\, h_1^2/2)_x+(q_{1,x}\, q_{1,y}/h_1)_y = -gh_1(h_2)_x+ gh_1\, H_x +S_{f1}(W)$$

$$(q_{1,y})_t+(q_{1,x}\, q_{1,y}/h_1)_x +(q_{1,y}^2/h_1 + g\, h_1^2/2)_y = -gh_1(h_2)_y+ gh_1\, H_y +S_{f2}(W)$$

$$(h_2)_t+(q_{2,x})_x+(q_{2,y})_y = 0$$

$$(q_{2,x})_t+(q_{2,x}^2/h_2 + g\, h_2^2/2)_x+(q_{2,x}\, q_{2,y}/h_2)_y = -grh_2(h_1)_x+ gh_2\, H_x +S_{f3}(W)+\tau_x$$

$$(q_{2,y})_t+(q_{2,x}\, q_{2,y}/h_2)_x+(q_{2,y}^2/h_2 + g\, h_2^2/2)_y = -grh_2(h_1)_y+ gh_2\, H_y +S_{f4}(W)+\tau_y$$

In these equations, index 1 corresponds to the upper layer and index 2 to the second layer. $h_i(x,y,t)$, i=1,2 is the layer thickness at each point (x,y) at time t, therefore h_2 stands for the thickness of the slide layer material; H(x,y) is the fixed bathymetry at (x,y) measured from a given reference level, $q_i(x,y,t)$, i=1,2 is the discharge and is related to the mean velocity by the equation $u_i(x,y,t)=q_i(x,y,t)/h_i(x,y,t)$, g is the gravitational acceleration and r is the ratio of densities $r=\rho_1/\rho_2$.

Terms $S_{fi}(W)$, i = 1, …, 4, model the different effects of the dynamical friction while $\boldsymbol{\tau} = (\tau_x, \tau_y)$ corresponds to the static Coulomb friction term. The terms S_{fi} are defined as follows:

$$S_{f1}(W) = S_{cx}(W) + S_{ax}(W), \quad S_{f2}(W) = S_{cy}(W) + S_{ay}(W),$$
$$S_{f3}(W) = -r\, S_{cx}(W) + S_{bx}(W), \quad S_{f4}(W) = -r\, S_{cy}(W) + S_{by}(W),$$

where $S_c(W) = (S_{cx}(W), S_{cy}(W))$ parametrizes the friction between layers and it is defined as follows:

$$S_{cx}(W) = m_f\, (h_1\, h_2)/(h_2 + r\, h_1)\, (u_{2,x} - u_{1,x})\, \|u_2 - u_1\|$$
$$S_{cy}(W) = m_f\, (h_1\, h_2)/(h_2 + r\, h_1)\, (u_{2,y} - u_{1,y})\, \|u_2 - u_1\|$$

being m_f a positive constant.

$S_a(W) = (S_{ax}(W), S_{ay}(W))$ parametrizes the friction between the water and the fixed bottom topography, if there is no granular material and it is defined by a Manning friction law:

$$S_{ax}(W) = -(g\, h_1\, n_1^2 / h_1^{(4/3)})\, u_{1,x}\, \|u_1\|$$
$$S_{ay}(W) = -(g\, h_1\, n_1^2 / h_1^{(4/3)})\, u_{1,y}\, \|u_1\|$$

where $n_1 > 0$ is the Manning coefficient, between the water and the fixed bottom topography.

$S_b(W) = (S_{bx}(W), S_{by}(W))$ parametrizes the dynamical friction between the debris layer and the fixed bottom topography and, as in the previous case, it is defined using a Manning law:

$$S_{bx}(W) = -(g\, h_2\, n_2^2 / h_2^{(4/3)})\, u_{2,x}\, \|u_2\|$$
$$S_{by}(W) = -(g\, h_2\, n_2^2 / h_2^{(4/3)})\, u_{2,y}\, \|u_2\|$$

where $n_2 > 0$ is the corresponding Manning coefficient.

Finally, $\boldsymbol{\tau} = (\tau_x, \tau_y)$ is the static Coulomb friction term and it is defined by

$$\text{if } \|\boldsymbol{\tau}\| \geq \sigma^c \Rightarrow \begin{cases} \tau_x = -g\,(1-r)\, h_2\, (q_{2,x} / \|q_2\|) \tan(\alpha) \\ \tau_y = -g\,(1-r)\, h_2\, (q_{2,y} / \|q_2\|) \tan(\alpha) \end{cases}$$

$$\text{if } \|\boldsymbol{\tau}\| < \sigma^c \Rightarrow \quad q_{2,x}=0,\ q_{2,y}=0,$$

where $\sigma^c = g\,(1\text{-}r)\,h_2 \tan(\alpha)$, being α the Coulomb friction angle. The above expression models the fact that a critical slope is needed to trigger the slide movement. It must be taken into account that the effects of hydroplaning may be important for submarine mass failures and tsunami generation highly reducing the expected value for σ^c.

The discretization of system (1) is performed by an explicit first-order IFCP Finite Volume Scheme where the discretization of the Coulomb friction term is performed following Fernández-Nieto et al. (2008) (see Fernández-Nieto et al. (2011) for details on the stability, convergence, and efficiency of the numerical scheme). The resulting scheme has been implemented in Graphics Processor Units (GPUs) using CUDA, achieving a speed-up of two orders of magnitude compared to a conventional CPU implementation (see Castro et al. (2011b) for a review and de la Asunción et al. (2012)). This methodology allows us to considerably improve the efficiency of the algorithm as well as the size of the discrete problems that can be solved.

This model reduces to the usual nonlinear shallow-water system when the layer of granular material is not present or when it has zero velocity and this layer reaches equilibrium. Therefore, the model can be used to numerically reproduce the different stages of a landslide tsunami simulation: the landslide tsunami generation, the far field wave propagation and, finally, the coastline inundation and the run-up height reached by the tsunami wave (Macías et al., 2015).

To perform BP#4 we have just implemented a dispersive version of the model above described and a version of a Savage-Hutter model coupled with a non-hydrostatic multilayer shallow-water system. One or both of these two models will be used to perform BP#4.

For BP#2 a non-hydrostatic multilayer shallow-water system has been implemented. The numerical model implementation combines finite-volume and finite-difference discretizations: the hyperbolic part of the model is discretized by means of a second-order path-conservative finite volume scheme. The elliptic problem associated to the non-hydrostatic terms are discretized using a second order central finite difference, and, finally, the time discretization is performed by means of a second order TVD Runge-Kutta scheme.

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